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Magneto-quantum oscillations in the Hall constant of three-dimensional metallic semiconductors

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Abstract. Magneto-quantum oscillations of the Hall constant have been observed in metallically doped three-dimensional semiconductors in a magnetic field far below the magnetic-field-induced metal–insulator (MI) transition. Especially around the magnetic field where all the electrons enter the lowest Landau level, a strong increase of the Hall constant is observed. This phenomenon is explained in terms of a field-induced MI transition in the tails of the spin-split Landau levels which can be understood in analogy to the well known MI transition in metallic semiconductors.

1. Introduction

In a three-dimensional (3D) metallic system where $\omega_c\tau \gg 1$ (ω_c is the cyclotron frequency and τ is the scattering time of the electrons), the transverse and longitudinal magnetoresistivities ρ_{xx} and ρ_{zz} are known to show magneto-quantum oscillations (Shubnikov–de Haas (SDH) effect) [1], but no pronounced structure is expected for the Hall resistivity ρ_{xy}. However, experimental observations of strong oscillations in the normalized Hall constant $R_H^N = \rho_{xy}/(B/ne)$ (n is the electron concentration) in 3D systems [2, 3] have attracted interest in view of the occurrence of plateaus in ρ_{xy} in two-dimensional electron gases (quantum Hall effect [4]).

Especially at a magnetic field where the electrons enter into the lowest spin-split Landau level a strong maximum appears in the Hall constant which is much bigger than the expected corrections due to higher-order scattering and admixtures of ρ_{xx} to ρ_{xy} of the order of $(\omega_c\tau)^{-2}$ [2]. Therefore, alternative mechanisms for the occurrence of this 0^- maximum‖ in the Hall constant have to be found.

Mani [3] has observed a plateau-like structure of the Hall resistivity in n-doped $Hg_{1-x}Cd_xTe$ and InSb. He explained this by supposing the existence of localized impurity states below each Landau level and he predicted *real* plateaus in the Hall resistivity. A recent theoretical model by Viehweger and Efetov [5] proposes that the Hall effect oscillations are due to the localization of the electrons in the tail of the spin-split Landau levels.

In this paper we will report on the experimental investigation of the Hall effect in metallically doped ($\omega_c\tau \gg 1$) III–V semiconductors (InSb and InAs). We observed

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‖ In the original notation [2, 11] the 0^+ level lies above the 0^- level. In this paper we follow the notation in [5] where the 0^+ level is situated below the 0^- level.

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magneto-quantum oscillations in the Hall constant, which saturate below 1 K without the occurrence of a real Hall plateau down to the lowest temperatures (40 mK). The deviation of ρ_{xy} from its classical value is especially strong (10% in InSb and 20% in InAs) at a magnetic field where the electrons enter into the lowest spin-split Landau level. In a given material its magnitude does not depend on $\omega_c\tau$ which shows that the observed effect is not due to any admixture of ρ_{xx}. Also the unexpectedly high magnitude of the last maximum in R_H^N and the fact that the maxima in ρ_{xx} and R_H^N occur at different magnetic fields confirm that the observed phenomena in R_H^N originate from the inherent properties of ρ_{xy}.

We will explain the occurrence of Hall effect oscillations using the above-mentioned theoretical model by Viehweger and Efetov [5]. Moreover, we will show quantitatively that these oscillations are due to a magnetic-field-induced metal–insulator transition [6] in the tails of the spin-split Landau levels.

2. Experimental details

We have investigated the transverse magnetoresistivity ρ_{xx} and the Hall resistivity ρ_{xy} using a standard phase-sensitive four-terminal AC method on four n-doped InSb samples (MCP Waver Technology Ltd, UK), and an n-doped InAs sample. The samples were spark cut from single crystals into the standard Hall bar geometry with four lateral contact legs and a size of about $10 \times 1 \times 2$ mm^3. Temperature ranges from 80 K down to 1.3 K in a ^{4}He cryostat and from 1.2 K down to 40 mK in a ^{3}He–^{4}He dilution refrigerator were explored, and magnetic fields up to 20 T were applied.

Table 1. Parameters of the measured samples (see text for explanation).

	InSb1	InSb2	InSb3	InSb4	InAs
n (10^{22} m^{-3})	1.6	6.3	5.1	5.4	2.7
μ (m^2 V^{-1} s^{-1})	7.8	6.4	6.8	6.8	4.3
B_{EQL} (T)	3.7	8.2	7.2	7.5	6.2
B_{MI} (T)	18	54	41	43	23
$\Delta n/n$ (%)	11	10	11	9	20
B_m (T)	3.25	7.9	6.6	6.8	5.8
δ	0.31	0.31	0.32	0.31	0.33

In order to study the semiconductors in their metallic state we have chosen high electron concentrations (table 1) well above the critical concentration n_c for a metal–insulator transition defined by the Mott criterion [6] $n_c^{1/3}a_B^* \simeq 0.25$, where a_B^* is the effective Bohr radius of the electrons. The experimentally determined electron concentrations n and the mobilities μ at 4.2 K are listed in table 1. All the samples have been strongly n doped with one single dopant (donors (Te or S) from group VI). Because of the high concentrations compensation effects have been neglected.

Metallically doped semiconductors with a high enough mobility μ ($\mu B = \omega_c\tau > 1$) exhibit magneto-quantum oscillations in ρ_{xx} due to the Landau level splitting of the electronic density of states up to the extreme quantum limit where all the electrons condense into the lowest Landau level. The magnetic field B_{EQL} where the extreme quantum limit begins has been determined from the point where the resistivity ρ_{xx} begins to increase monotonically (table 1).

Additionally, the decrease of the extension of the electronic wave function in a strong magnetic field will induce a metal–insulator transition. In a strong magnetic field where

$l_B \ll a_B^*$ ($l_B = \sqrt{\hbar/eB}$ is the magnetic length) the electronic wave function can be described as an ellipsoid with extensions $a_\parallel = a_B^*/\ln(a_B^*/l_B)^2$ parallel to the field and $a_\perp = 2l_B$ perpendicular to B. The transition field B_{MI} can then be defined in analogy to the above-described Mott criterion for an MI transition at zero field by the magnetic freeze-out condition [7]

$$(na_\parallel a_\perp^2)^{1/3} \simeq 0.3\,. \tag{1}$$

The calculated transition fields B_{MI} for our samples are listed in table 1. For the calculation of the Bohr radius a_B^* in a magnetic field the field dependence of the effective mass at the Fermi level has to be taken into account. In a high magnetic field where only the lowest Landau level is occupied this can be found implicitly from the energy dependence of the effective mass in the two-band model [8]

$$m^*(E) \simeq m_0^*\left(1 + \frac{2E}{\Delta}\right) \tag{2}$$

(Δ is the energy gap and m_0^* is the effective mass at the bottom of the band) by taking [1]

$$E = \frac{1-\nu}{2}\hbar\omega_c + \frac{16}{9}\frac{E_F^3}{(\hbar\omega_c)^2} \tag{3}$$

(ν is the spin splitting factor and E_F is the Fermi energy at zero field) as the Fermi energy in the extreme quantum limit. Using $m_0^* = 0.014m_e$ and $\Delta = 0.235$ eV for InSb ($m_0^* = 0.023m_e$ and $\Delta = 0.418$ eV for InAs) we find that $l_B \ll a_B^*$ is well fulfilled for the calculated transition fields of all the samples.

Since the condition $B_{EQL} \ll B_{MI}$ is verified in all the investigated samples, the quantum oscillation regime and metal–insulator transition are well separated. We can therefore state that, up to B_{EQL} and even above, all the samples can be regarded as good metals, i.e. $\omega_c\tau = \rho_{xy}/\rho_{xx} \gg 1$.

3. Results

A typical result (sample InSb1) for the experimentally measured quantities ρ_{xx} and ρ_{xy} is plotted in figure 1(a). The sample shows metallic behaviour ($\rho_{xy}/\rho_{xx} \gg 1$) for all magnetic fields up to the extreme quantum limit and well above which is also the case for all the other samples. ρ_{xy} is typically one order of magnitude larger than ρ_{xx}. The minimal value of ρ_{xy}/ρ_{xx} is between five (InSb1 and InAs) and 10 (InSb2, InSb3, and InSb4) at the magnetic field where the last SDH maximum occurs in ρ_{xx}.

The transverse magnetoresistivity ρ_{xx} shows strong SDH oscillations which are smeared out by temperature and level broadening. For temperatures lower than 1 K, down to 40 mK, and for all the samples, the temperature broadening of the Landau levels can be neglected as compared to the natural level broadening and, therefore, the oscillatory part of ρ_{xx} does not change any more. (A logarithmic decrease of ρ_{xx} in the extreme quantum limit below 1 K has been reported elsewhere [9].)

In a first approximation ρ_{xy} can be described by its classical value $\rho_{xy}^{cl} = B/ne$. However, there are slight magneto-quantum oscillations superimposed on this behaviour which will be discussed in detail below. To check that these effects are not related to a geometric artifact, we have measured ρ_{xy} also in a reversed magnetic field. ρ_{xy}, and, in particular, its strong structure around B_{EQL}, was verified to change sign when reversing the field. Therefore we can be sure that the observed effects are not mainly due to an admixture of ρ_{xx} and ρ_{xy} coming from a misalignment of the contact legs where the Hall voltage was

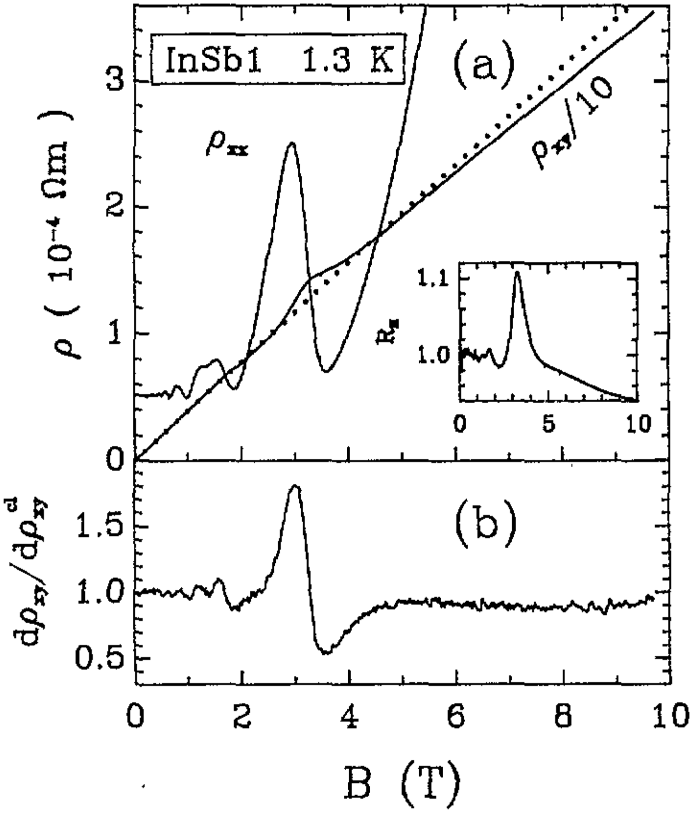

Figure 1. (a) Transverse magnetoresistivity ρ_{xx} and Hall resistivity ρ_{xy} of the sample InSb1 at 1.3 K. The dotted line represents the classical value $\rho_{xy}^{cl} = B/ne$ and the inset shows the normalized Hall constant $R_H^N = \rho_{xy}/\rho_{xy}^{cl}$. (b) Derivative of ρ_{xy} with respect to ρ_{xy}^{cl} at the same temperature.

measured or from a slight tilting of the sample with respect to the plane perpendicular to the field. We have also checked the homogeneity of the electron concentration by measuring the Hall voltage at different pairs of contact legs and obtaining the same results. As a conclusion of these verifications we are able to estimate that ρ_{xy} reflects within less than 1% its own inherent properties.

In the extreme quantum limit, before reaching the metal–insulator transition field B_{MI}, the normalized Hall constant R_H^N undershoots its classical value of unity (inset in figure 1(a)). The occurrence of this 'Hall dip' has been explained by the beginning of localization of electrons on shallow donors [10]. Regions of the sample with donors having no neighbouring donor site within the distance of the extension of the electronic wave function already become insulating below B_{MI}. Since the transition criterion [7] $(na_\parallel a_\perp^2)^{1/3} \simeq 0.3$ is not yet fulfilled globally in this field range metallic clusters with a higher electron concentration n_{eff} than the average concentration n are formed in the remaining parts of the crystal. This higher effective concentration leads to a considerable reduction of the normalized Hall constant $R_H^N = n/n_{eff}$ as compared to its classical value of unity. Approaching the field B_{MI}, the total free electron concentration decreases, and, therefore, R_H^N will increase again.

However at lower fields around and below B_{EQL} the Hall resistivity also deviates from the classically predicted straight line. Especially around the magnetic field where the electrons enter into the lowest Landau level a strong plateau-like structure of ρ_{xy} can be observed. This structure is shown in detail in figure 1(b) where the derivative of ρ_{xy} with respect to its classically expected value $\rho_{xy}^{\mathrm{cl}} = B/ne$ is displayed. At the plateau-like structure of ρ_{xy} around B_{EQL} the slope $\mathrm{d}\rho_{xy}/\mathrm{d}\rho_{xy}^{\mathrm{cl}}$ only decreases to about half of its classical value and does not reach zero as is the case for the real plateaus in the 2D quantum Hall effect. This means that also at the field where the structure in ρ_{xy} develops, there are still enough free electron states at the Fermi level, which is also the very reason why the minima of ρ_{xx} have a non-zero value even for $T \to 0$. Since below 1 K the structure in ρ_{xy} is not dependent on T, the non-existence of a real plateau could be verified experimentally down to the lowest temperatures (40 mK).

For a clearer visualization of the Hall effect oscillations we have plotted the normalized Hall constant $R_{\mathrm{H}}^{\mathrm{N}} = \rho_{xy}/\rho_{xy}^{\mathrm{cl}}$ of the sample InSb4 in figure 2(a) and compare it with the SDH oscillations $\rho_{xx}/\rho_{xx}^{\mathrm{cl}}$ in figure 2(b). To compensate for classical magnetoresistance effects, ρ_{xx}^{cl} has been determined from the high-temperature data at 80 K, and corrected by a slight temperature dependence of the electron concentration of the order of 2% between 4 K and 80 K.

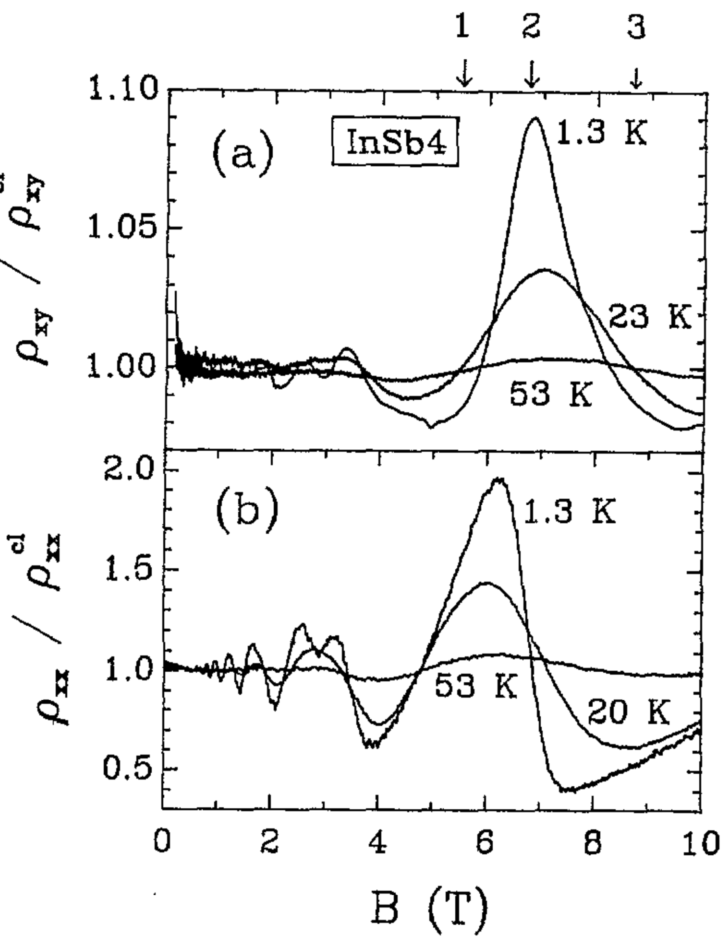

Figure 2. Magneto-quantum oscillations in ρ_{xy} (a) and ρ_{xx} (b) of the sample InSb4 for different temperatures. The arrows mark magnetic fields corresponding to different positions of the Fermi level as shown in figure 3.

As in ρ_{xx}, magneto-quantum oscillations are also visible in R_H^N. However, the higher-order maxima in R_H^N are much more strongly suppressed than the corresponding SDH peaks, which leads us to the conclusion that the magnetic field where the electrons enter the lowest Landau level plays a distinct role for the Hall effect. The maxima in R_H^N are slightly shifted towards higher fields as compared to the maxima in ρ_{xx}. As a maximal ρ_{xx} corresponds to a situation where the Fermi energy is situated at the energy with a maximal density of states in a Landau level, the Hall constant develops its maxima where the Fermi level has already crossed this Landau level of which only the low-energy tail remains occupied.

An oscillatory behaviour of the Hall constant shifted with respect to the SDH oscillations of ρ_{xx} and a pronounced last maximum was also observed in the three other InSb samples (with a maximal deviation from the classical value of about 10%) and in the InAs sample (20% maximal deviation).

4. Discussion

A recent theoretical model [5] considers the situation with the two lowest spin-split Landau levels 0^+ and 0^- being occupied (figure 3). The density of states of each of these two Landau levels can be decomposed into two regions: the low-energy tails (shaded areas in figure 3) where the electrons are localized and the extended electron states for electron energies above. It is known that the two spin states can be regarded as two separated subsystems with electron concentrations n^+ and n^- with no inter-level scattering [11] which has been confirmed experimentally by the absence of a 0^- Shubnikov–de Haas maximum in the longitudinal resistivity ρ_{zz} [2].

When both levels are nearly completely occupied (Fermi level at position 1 in figure 3) all the electrons states are free and the Hall resistivity has its classical value, slightly modified by corrections of the order of at most $(\omega_c\tau)^{-2}$.

Increasing the magnetic field the number of free electrons decreases continuously when the Fermi level approaches position 2, because the free states in the 0^- level move gradually above the Fermi level and the remaining states in the tail become more and more localized. At position 2 all the 0^- electrons are localized in the tail. Since in a further increasing magnetic field the 0^- band becomes more and more depopulated the localized electrons reappear shortly after position 2 in the free 0^+ states at position 3. The observed increase of the free electron concentration between position 2 and position 3 is thus equal to the number of localized electrons in the tail of the 0^- level at position 2. This prediction is quantitatively proven by the experimentally observed decrease of $R_H^N = \rho_{xy}/\rho_{xy}^{cl}$ (i.e. an increase of n_{free}) above the field B_m where R_H^N reaches its maximum.

In order to verify quantitatively the observations concerning the structure in ρ_{xy} around B_m and to find an estimation for the degree of localization in the tail of the 0^- Landau level, we have listed in table 1 the maximal change in the free electron concentration determined from the change in ρ_{xy} at B_m as well as the magnetic field B_m where this occurs. We propose that the localization can be attributed to the fact that the electrons of the 0^- subsystem with a concentration n^- have undergone a magnetic-field-induced metal–insulator transition. This can be confirmed by calculating the parameter $\delta = (a_\parallel a_\perp^2 n^-)^{1/3}$ [7] which has already been introduced in equation (1) where the 'normal' field-induced metal–insulator transition has been described. The obtained values for δ listed in table 1 are close to the theoretically expected value of about 0.3 for all the samples. The field where the strong maximum in R_H^N occurs is then the metal–insulator transition field for the 0^- system.

Normally metal–insulator transitions are characterized by a drastic change of the

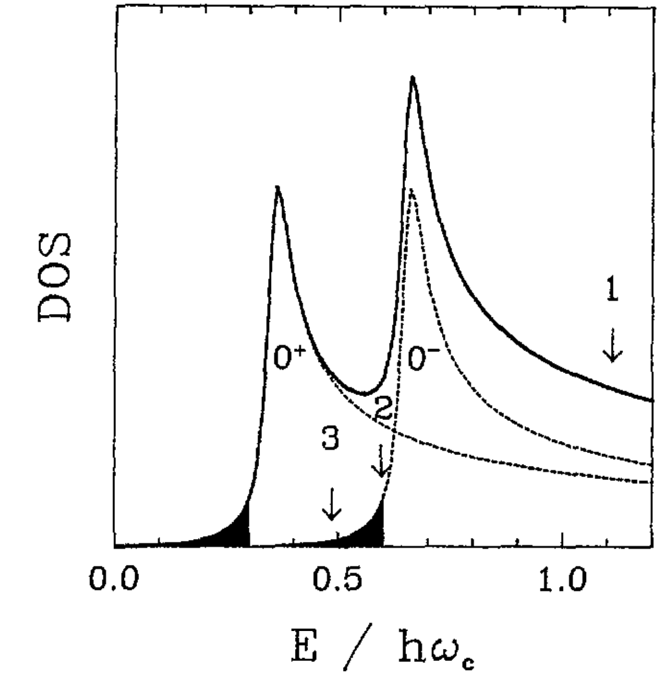

Figure 3. Schematic representation of the two lowest spin split Landau levels 0^+ and 0^- (dashed lines), with shaded regions for the localized states. The solid line represents the total density of states. The arrows mark different positions of the Fermi energy numbered for increasing magnetic fields (see text).

resistivity at the transition field [7]. However, we only observe an effect of about 10% in R_H because the resistivity and the Hall constant are mainly determined by the metallic 0^+ subsystem. The field where the maximum occurs can therefore be regarded as the start of the metal–insulator transition in the 0^- subsystem. Without the presence of the dominating 0^+ system this field can be related to a drastic increase of R_H of the 0^- system.

For fields below B_m, R_H^N undershoots its classical value. For the presented data this effect is about 2% at a field where $(\omega_c\tau)^{-2}$ is smaller than 3×10^{-3}, so again effects due to scattering can be neglected. A lower Hall constant is known to be observed for fields below a field-induced metal–insulator transition ('Hall dip') [10]. This effect has already been mentioned above to explain the decrease of R_H^N in the extreme quantum limit below B_{MI} (see figure 1(a)). The undershooting of R_H^N below B_m could correspond to the 'Hall dip' of the metal–insulator transition of the 0^- states to which the increase of R_H^N up to B_m has been attributed.

We can also observe higher-order maxima in R_H^N corresponding to the situation where several Landau levels are still occupied. However, they are much less pronounced than the corresponding SDH maxima. To understand this point, one should remember that the Hall constant maxima occur at magnetic fields where only the tail of a Landau level is occupied. Only the very few electrons in this tail (as compared to the total number of free electrons in several Landau levels) contribute to the change of the Hall constant. Moreover, as now also extended states with the same spin as the tail states exist, these states can now mix with the tail states [5]. On the other hand, the SDH effect observed in ρ_{xx} is sensitive to the local density of states around the Fermi level which makes the higher-order maxima in ρ_{xx}

relatively more pronounced than the corresponding maxima in R_{H}^{N}.

5. Conclusions

In summary, we have shown that the strong deviation of the Hall constant from its constant classical value in metallically doped semiconductors at magnetic fields where the electrons enter the lowest Landau level is due to a field-induced metal–insulator transition of the electrons in the tail of the spin-split Landau level above. No other localization mechanism had to be assumed, and, as expected, no real Hall plateau shows up in the Hall resistivity as is the case in two-dimensional systems.

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