ELSEVIER

Physica B 204 (1995) 90–94

PHYSICA B

# Extrinsic magnetoresistance in semiconductors

U. Zeitler[a,b,*], A.G.M. Jansen[a]

[a] *High Magnetic Field Laboratory, Max-Planck-Institut für Festkörperforschung/Centre National de la Recherche Scientifique, B.P. 166, F-38042 Grenoble Cedex 9, France*

[b] *High Field Magnet Laboratory, University of Nijmegen, Toernooiveld 1, NL-6525 ED Nijmegen, The Netherlands*

**Abstract**

The extrinsic magnetoresistance of a metallically doped InSb sample has been investigated. At high temperatures, where no magnetoquantum oscillations occur, an additional linear magnetoresistance (LMR) arising from a difference in the Hall voltages can be observed. This effect is shown to be due to macroscopic variations of the electron concentration and of the thickness. At low temperatures these two contributions can be measured independently with the help of the superimposed magnetoquantum oscillations.

## 1. Introduction

Since the first detailed investigations of Kapitza [1] the magnetoresistance of metals has been studied intensively [2]. Besides a quadratic magnetoresistance at low magnetic fields and magnetoquantum oscillations no magnetoresistance is expected in the classical high field limit ($\omega_c\tau \gg 1$, $\omega_c$ is the cyclotron frequency and $\tau$ the scattering time of the electrons) where only one type of carrier exists. Therefore, an experimentally observed linear magnetoresistance (LMR) in simple metals like aluminium has been puzzling the scientific community for a long time and the question whether this effect is rather intrinsic or extrinsic remained open.

An important part of the LMR in a transverse geometry, where a current is flowing in the direction perpendicular to the applied magnetic field, could be attributed to thickness variations of a sample parallel to the field [3]. These thickness variations lead to different Hall voltages in different parts of the sample which have to be compensated by an additional voltage drop along the current direction.

Apart from metals, extrinsic magnetoresistance effects also occur in strongly doped semiconductors like InSb or InAs [4]. Due to the low effective mass in such systems the condition $\omega_c\tau \gg 1$, which is necessary to observe a strong LMR, can be easily fulfilled, even at room temperature. Furthermore, due to the low electron concentration, the absolute value of LMR which is proportional to the Hall voltage is several orders of magnitude higher than in normal metals. It can therefore be easily measured with standard phase-sensitive AC-detection (signals of a few m$\Omega$) whereas in metals normally only very low DC-signals (a few n$\Omega$) can be observed.

In this paper we will report on the magnetoresistance in InSb. We will show that the observed LMR arises from two contributions, namely variations of the thickness perpendicular to the current direction and macroscopic fluctuations of the electron concentration. The LMR of a stepped sample at low temperatures will be shown being proportional to the Hall voltage *difference* between the voltage probes where the resistance is measured.

* Corresponding author.


*SSDI* 0921-4526(94)00247-9

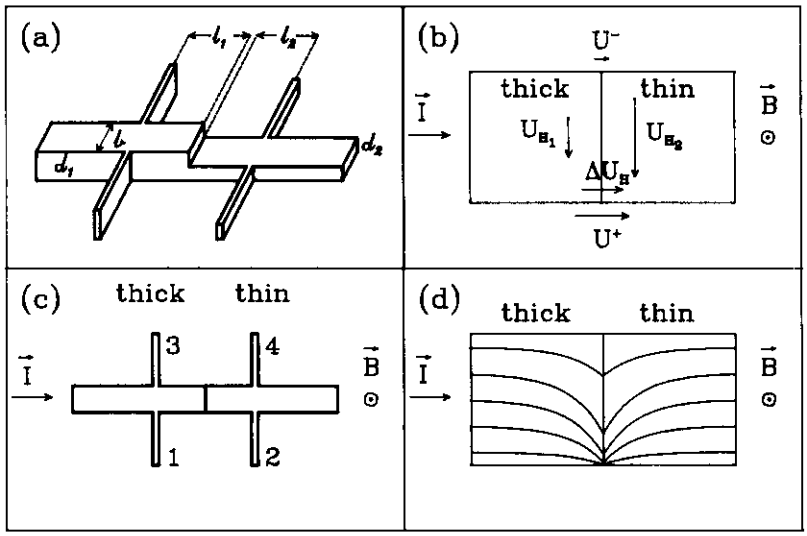


Fig. 1. (a) Geometry of a stepped sample where a strong LMR occurs. (b) Voltage drops in the sample (top view). (c) Designation of the contacts. (d) Schematic current distribution for $\tan \vartheta = 2$.

## 2. Models

The phenomenon of an extrinsic LMR is illustrated in Fig. 1. Suppose a simple geometry of an edge-shaped sample having a uniform Hall resistivity $\rho_{xy} = B/ne$ ($n$ is the electron concentration) with a thick part (thickness $d_1$) and a thin part (thickness $d_2$) where a total current $I$ is injected. The Hall voltages in the two parts are $U_{H_1} = I\rho_{xy}/d_1$ and $U_{H_2} = I\rho_{xy}/d_2$, respectively. Since in a stationary case ($dB/dt = 0$) the total voltage drop over a closed path is zero, an additional voltage $U_{H_2} - U_{H_1}$ has to be built up along one side of the sample (Fig. 1(b)).

There exist mainly two models which explain the extrinsic contribution to the LMR of metals. The first is based on traditional electrodynamics [5]. Solving Maxwell's equations with appropriate boundary conditions leads to an explicit expression for the extrinsic effects on the magnetoresistance of a metal (or a metallic semiconductor).

A second theory is based on the so-called transmission approach which has initially been developed for ballistic transport in mesoscopic systems and in two-dimensional electron gases where the quantum Hall effect occurs [6] and then extended to the case of a macroscopic three-dimensional metal [7]. In this model the skipping-orbits along the edges of the sample are only partly transmitted on a step in the thickness.

Without going into the details of the theory we can summarize the main results [5, 7]. The experimentally measured voltages $U^+$ and $U^-$ on two contact-leg pairs 1–2 and 3–4, respectively, (Fig. 1(c)) can be given by

$$U^+ = U_0 + \Delta U_H - U_1(B) \tag{1}$$

and

$$U^- = U_0 - U_1(B). \tag{2}$$

This magnetoresistance contains three terms, namely

$$U_0 = \frac{\rho_{xx} I}{b}\left(\frac{l_1}{d_1} + \frac{l_2}{d_2}\right) \tag{3}$$

(the quantities $b$, $l_1$, $l_2$, $d_1$ and $d_2$ for the sample dimensions are defined in Fig. 1) which is the contribution arising from the intrinsic resistance of the sample and

$$\Delta U_H = U^+ - U^- = \rho_{xy} I\left(\frac{1}{d_2} - \frac{1}{d_1}\right) \tag{4}$$

being the difference of the Hall voltages in the two parts of the sample which has to be compensated. In an inverted magnetic field this LMR occurs on the other sample side, i.e. the voltages $U^+$ and $U^-$ are interchanged.

The additional correction $U_1(B)$ which arises on both sides of the sample saturates for high fields ($\omega_c\tau \gg 1$) to a value of the order of $(b/l)U_0$ and can be neglected in the case where the distance $l$ between the voltage probes is large as compared to the sample width $b$.

## 3. Experimental results and discussion

We have studied LMR effects on an n-doped InSb sample with an electron concentration $n \approx 5.4 \times 10^{22}\ \mathrm{m}^{-3}$ and a mobility of $\mu \approx 6.8\ \mathrm{m^2V^{-1}s^{-1}}$ (at 4.2 K). The sample was spark cut in the standard Hall bar geometry with the same thickness of 1.7 mm along the x-axis (further on referred to as flat sample). Later, the thickness in half of the sample has been reduced to about 1.4 mm to get the form shown in Fig. 1(a) (stepped sample).

### 3.1. The classical regime

In order to separate extrinsic effects clearly from intrinsic magnetoquantum oscillations, the flat sample has first been measured at 77 K where no oscillating quantum effects occur. The results for the two transverse resistances $R_{12}$ (defined by the voltage measured between the contact legs 1 and 2 divided by the applied current) and $R_{24}$ and the two Hall resistances $R_{13}$ and $R_{34}$ are shown in Fig. 2. For the definition of the contact legs we refer to Fig. 1(c). (For the moment a flat sample is considered, i.e. $d_1 = d_2$).

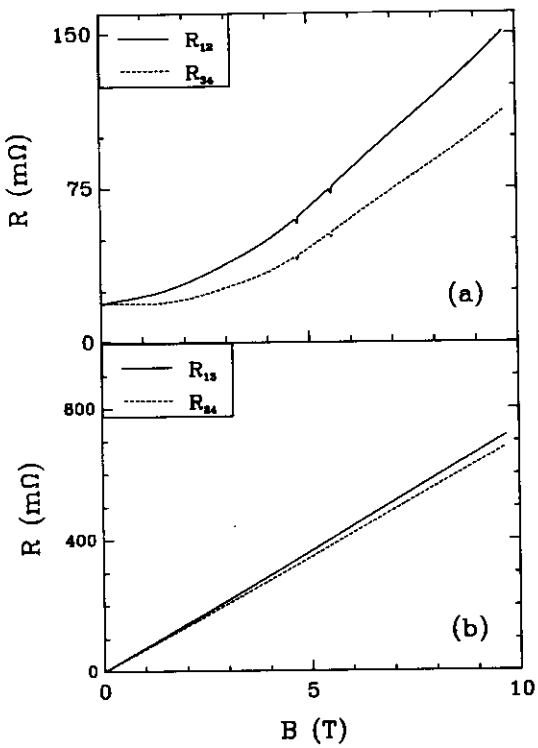


Fig. 2. The transverse resistance $R_{12}$ and $R_{34}$ (a) and the Hall resistances $R_{13}$ and $R_{24}$ (b) in the flat sample at 77 K.

As can be seen in Fig. 2(b), the Hall voltages measured on two different contact-leg pairs (1–3 and 2–4) differ by about 5%. This difference has to be mainly due to a different electron concentration. To produce the same magnitude with a step, a thickness variation of about 0.1 mm would be necessary. Already from this result we can estimate that the macroscopic concentration fluctuations in the sample on a length scale defined by the contact-leg distance are of the order of a few percent.

However, as we deal with a good metal ($\rho_{xy} \gg \rho_{xx}$, i.e. $\omega_c\tau \gg 1$), such a small variation in the Hall voltage can nevertheless lead to an appreciable supplementary LMR on either $R_{12}$ or $R_{34}$ (depending on the orientation of the magnetic field). This observation is shown in Fig. 2(a). The resistance $R_{12}$ on one side of the sample is in fact considerably larger than $R_{34}$ on the other side. The difference $\Delta R = R_{12} - R_{34}$ agrees with the difference of the Hall voltages $\Delta R_H = R_{13} - R_{24}$.

We have also verified that in an inverted magnetic field the additional LMR is observed on the opposite side. The expected relations

$$R_{12}^{+} = R_{34}^{-}, \qquad R_{34}^{+} = R_{12}^{-}, \tag{5}$$

$$R_{13}^{+} = -R_{13}^{+}, \quad R_{24}^{+} = -R_{24}^{-}$$

are well fulfilled. Here $R_{ij}^{\pm}$ is the resistance between $i$ and $j$ in a magnetic field pointing in the $\pm z$-direction, respectively.

To conclude, it can be stated that the LMR which occurs in metallically doped semiconductors can be used as a method to determine macroscopic concentration fluctuations in such a system [4]. The length scale on which this can be done is defined by the distance of the voltage probes and the relative accuracy is given by the relative flatness of the sample on this length scale.

### 3.2. The magnetoquantum-oscillatory regime

The influence of a geometrically induced linear magnetoresistance (LMR), i.e. a magnetoresistance proportional to a difference of two Hall voltages, can be particularly interesting when also the intrinsic resistance shows a pronounced behaviour which depends, as the LMR, on the electron concentration. Superimposing these two contributions allows us to distinguish clearly between a LMR which is due to variations either of the sample thickness or of the electron concentration.

To investigate such effects we have reduce the thickness of the sample on half of its length to about 1.4 mm (80% of the original thickness) and measured the resistances $R_{12}$ and $R_{34}$ as well as the Hall resistances $R_{13}$ (thick side) and $R_{24}$ (thin side) at liquid-helium temperature (4.2 K). The results are plotted in Fig. 3. For the

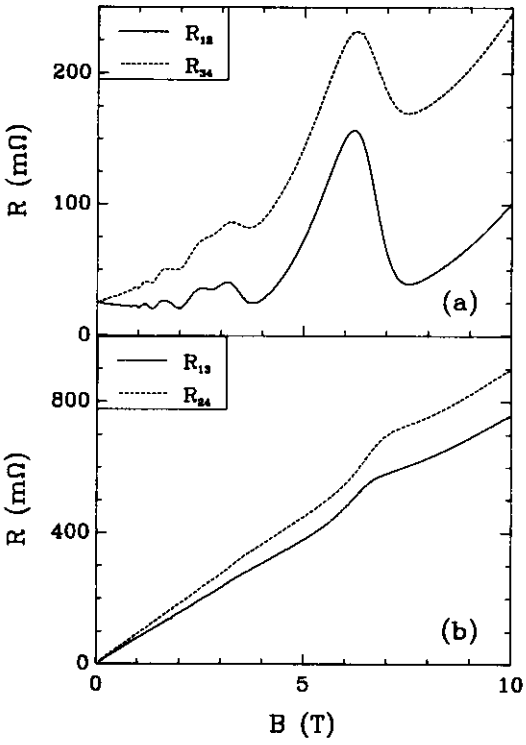


Fig. 3. The transverse resistances $R_{12}$ and $R_{34}$ (a) and the Hall resistances $R_{13}$ and $R_{24}$ (b) in the stepped sample at 4.2 K.

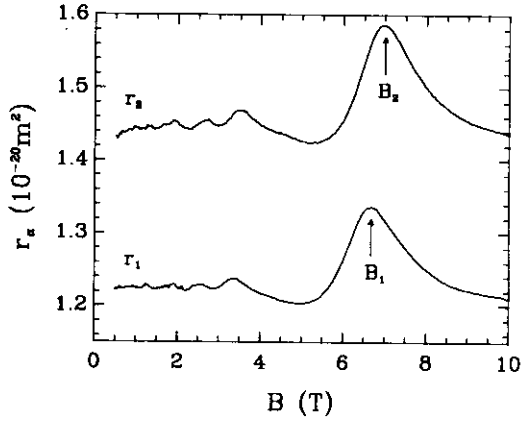


Fig. 4. The quantities $r_\alpha = 1/d_\alpha n_\alpha$ in the thick part ($\alpha = 1$) and in the thin part ($\alpha = 2$) of the stepped sample at 4.2 K.

definition of the contact-leg numbers we refer again to Fig. 1(c). The designation of the contact legs as well as the orientation of the magnetic field are the same as for the flat sample in Fig. 2.

We know already from above that the electron concentration $n_1$ between the contact legs 1 and 3 in the thick part is about 5% smaller than $n_2$ between 2 and 4 in the thin part. To confirm this observation we will first have a look at the quantities $r_\alpha \equiv eR_{ij}/B$ with $\alpha = 1$ for the contact-leg pair (13) in the thick part and $\alpha = 2$ for the pair (24) in the thin part. If we suppose a classical Hall resistivity $\rho_{xy} = B/en_\alpha$ in both parts of the sample we can write $r_\alpha = 1/n_\alpha d_\alpha$, where $d_\alpha$ is the sample thickness in the part concerned. $r_\alpha$ therefore only contains the thickness of the sample at the place where the Hall voltages are measured and the corresponding electron concentration.

On both sides $r_\alpha$ contains a constant background $r_\alpha^0$ onto which magneto-quantum oscillations are superimposed (Fig. 4). These oscillations were already analysed in detail in another paper [8]. The last pronounced peak in $r_\alpha$ occurs at a magnetic field where all the electrons enter into the lowest spin-split Landau level. This phenomenon can be understood in terms of a magnetic-field induced metal–insulator transition which occurs in the tail of the second lowest Landau level. Since the degeneracy of a Landau level is proportional to the electron concentration $n$ also the position of $B_\alpha$ depends linearly on the electron concentration at the corresponding contact. Therefore, we have two equations which determine fully the ratios $n_1/n_2$ and $d_1/d_2$ namely

$$\frac{n_1}{n_2} = \frac{B_1}{B_2}, \quad \frac{n_1 d_1}{n_2 d_2} = \frac{r_2^0}{r_1^0}. \tag{6}$$

From $B_1 = 6.67$ T and $B_2 = 7$ T we can first confirm independently from above the statement $n_1 = 0.95 n_2$. A difference in the Hall effect arising from this concentration difference will therefore already give rise to a LMR on one side of the sample. Since we have moreover $d_2 \approx 0.8 d_1$ a further Hall voltage difference (in the opposite direction) occurs. The sum of these two differences is represented in Fig. 5. As can be seen the additional magnetoresistance $R_{34} - R_{12}$ corresponds exactly to the difference of the Hall resistances $R_{24} - R_{13}$. The structure in the Hall effect which appears at different fields in the two parts of the sample is strongly enhanced in the additional contribution to the magnetoresistance.

These experimental results open a new method to investigate precisely small structures in the Hall resistivity which depend on the electron concentration. A sample could be grown in such a way that two different parts with different concentrations $n_1$ and $n_2$ exist. The thicknesses of the two parts can then be chosen such that $d_1 n_1 = d_2 n_2$. A magnetic-field and

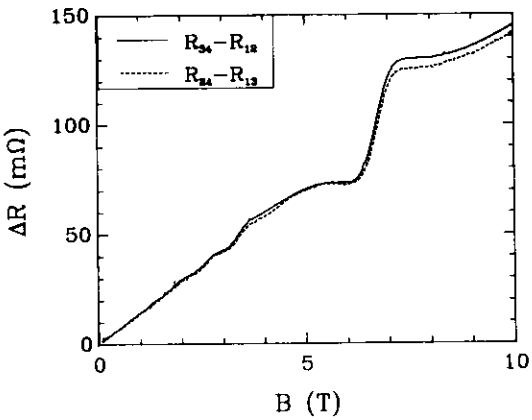


Fig. 5. The difference of transverse resistances $R_{34} - R_{12}$ and of the Hall resistances $R_{24} - R_{13}$ in the stepped sample at 4.2 K.

concentration-dependent small structure $\delta(n, B) \ll 1$ in the Hall resistivity

$$\rho_{xy} = \frac{B}{ne}(1 + \delta(n, B)) \tag{7}$$

can now be measured accurately by using the LMR effect. The additional magnetoresistance of one side of the sample will be given by

$$\Delta R(B) = \frac{B}{n_1 d_1 e}|\delta(n_1, B) - \delta(n_2, B)|. \tag{8}$$

Of course the same information can, in principle, be obtained by measuring the two Hall voltages separately instead of measuring the voltages $U^+$ and $U^-$ along two sides of the sample. But since the intrinsic resistance in the high field case ($\omega_c\tau \gg 1$) is a factor of $\omega_c\tau$ lower than the Hall resistance, the relative effects on $U^+$ (or $U^-$, depending on the sign of $B$) are a factor of $\omega_c\tau$ higher than on the individually measured Hall voltages. In other words, changes in $\rho_{xy}$ can be observed on a scale of $\rho_{xx}$ in systems where $\rho_{xy} \gg \rho_{xx}$. Furthermore, the LMR only contains the difference of two Hall voltages without any superimposed geometric admixture of $\rho_{xx}$.

## 4. Conclusions

In summary we have shown that the "linear magnetoresistance" in metallic semiconductors can be used to measure small differences of the Hall voltage at two different contact legs. These differences can be either due to geometric thickness variations or to variations in the macroscopic charge carrier concentration. Not only a linear contribution is observed but also intrinsic properties of the Hall effect can be investigated with this method without the need of a Hall bar geometry on the contact legs.

## References

[1] P. Kapitza, Proc. Roy. Soc. A 123 (1929) 292.
[2] See, for example, A.B. Pippard, Magnetoresistance of Metals (Cambridge University Press, Cambridge, 1989).
[3] G.J.C.L. Bruls, J. Bass, A.P. van Gelder, H. van Kempen and P. Wyder, Phys. Rev. Lett. 46 (1981) 553; G.J.C.L. Bruls, J. Bass, A.P. van Gelder, H. van Kempen and P. Wyder, Phys. Rev. B 32 (1985) 1927.
[4] R.T. Bate and A.C. Beer, J. Appl. Phys. 32 (1961) 800; R.T. Bate, J.C. Bell and A.C. Beer, J. Appl. Phys. 32 (1961) 806; R.T. Bate, in: Semiconductors and Semimetals, Vol. 24, eds. R.K. Willardson and A.C. Beer (Academic Press, New York, 1968).
[5] G.L.J.A. Rikken, J.A.M.M. van Haaren, A.P. van Gelder, H. van Kempen, P. Wyder, H.-U. Habermeier and K. Ploog, Phys. Rev. B 37 (1988) 10 229.
[6] A.H. MacDonald and P. Středa, Phys. Rev. B 29 (1984) 1616; M. Büttiker, Phys. Rev. B 38 (1988) 9375.
[7] M. Büttiker, Phys. Rev. B 42 (1990) 3197; P. Scheuzger, A.G.M. Jansen, P. Wyder, P.A. Schroeder, M. Büttiker and G. Bruls, Phys. Rev. B 47 (1993) 3779.
[8] U. Zeitler, A.G.M. Jansen, P. Wyder and S.S. Murzin, J. Phys.: Condens. Matter 6 (1994) 4289.