High frequency conductivity in the quantum Hall effect

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Abstract

We present high frequency measurements of the diagonal conductivity σ_{xx} of a two-dimensional electron system in the integer quantum Hall regime. The width of the σ_{xx} peaks between QHE minima is analyzed within the framework of scaling theory using both temperature ($T = 100$–700 mK) and frequency ($f \leqslant 6$ GHz) in a two parameter scaling ansatz. For the plateau transition width Δv we find scaling behaviour for both its temperature dependence as well as its frequency dependence. However, the corresponding scaling exponent for temperature ($\kappa = 0.42$) significantly differs from the one deduced for frequency scaling ($c = 0.6$). Additionally, we use the high frequency experiments to suppress the contact resistances that strongly influences DC measurements. We find an intrinsic critical conductivity $\sigma_c = 0.17e^2/h$, virtually independent of temperature and filling factor, and deviating significantly from the proposed universal value $0.5e^2/h$. © 2001 Elsevier Science B.V. All rights reserved.

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Fundamental progress in the understanding of the properties of a two-dimensional electron system (2DES) in high magnetic fields was brought by the application of scaling theory to the transition between quantum Hall plateaus (review [1] and references therein). Within this theoretical picture the step in the Hall conductivity σ_{xy} between quantized values and the corresponding maximum of the longitudinal conductivity σ_{xx} are governed by a diverging localization length $\xi \propto |v - v_c|^{-\gamma}$ with filling factor v and critical point v_c near half integer filling. The exponent γ is believed to be universal with a value of $\gamma = 2.3$ found in numerical studies and size scaling experiments. The conductivities are given by scaling functions $\sigma_{\alpha\beta}(L_{eff}/\xi)$. L_{eff} is an effective length scale governed by sample size, temperature or frequency. The natural temperature dependent length scale is determined by the phase coherence length $L_\Phi \propto T^{-p/2}$ with $p = 2$ deduced from the experiment. Applying a high frequency to the system introduces an additional length scale, the dynamic length $L_f \propto f^{-1/z}$ with dynamic exponent $z = 1$ found from numerical studies [2]. As a consequence of the length scales defined above, the width Δv of the conductivity peak around v_c is predicted to follow power laws, $\Delta v(T) \propto T^\kappa$ with $\kappa = p/2\gamma$ and $\Delta v(f) \propto f^c$ with $c = 1/z\gamma$. The temperature dependence has been subject to many experiments, mostly with the result of $\kappa \approx 0.43$ from which follows $p = 2$. Only few experiments [3–5] addressed the frequency dependence and yielded contradicting results: While Engel et al. [3] measured scaling behaviour with $c \approx 0.43$ consistent

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with $z = 1$, an experiment of Balaban et al. [4] contradicts scaling.

Right at the critical point σ_{xx} has its maximum value and is named critical conductivity σ_c. Following scaling theory this value should be independent of temperature, frequency and of the viewed transition identified by v_c. Even further there are analytical arguments and numerical calculations for a sample independent universal critical conductivity value of $\sigma_c = 0.5e^2/h$ (references in Ref. [1]), but most experiments do not even follow the first prediction.

In this paper, we report frequency and temperature dependent measurements with $f = 100$ kHz–6 GHz and $T = 100$–700 mK which are analyzed within the framework of scaling theory. The measured transition widths follow scaling behaviour and are analyzed using a two parameter scaling ansatz. We find different exponents $\kappa = 0.42 \pm 0.05$ and $c = 0.6 \pm 0.1$ for frequency and temperature dependence.

The critical conductivities σ_c at low frequencies $f < 1$ GHz are temperature, frequency and transition dependent which can be understood by contact effects. At high frequency contact effects are negligible and we measure a transition independent non-universal value $\sigma_c = 0.17e^2/h$.

The sample used in the present work is an Al-GaAs/GaAs heterostructure grown by molecular-beam epitaxy containing a 2DES with a moderate electron mobility $\mu_e = 35$ m^2/V s and an electron density $n_e = 3.3 \times 10^{15}$ m^{-2}. The sample was patterned in a Corbino geometry with ohmic contacts fabricated by standard Ni/Au/Ge alloy annealing (see inset in Fig. 1). The Corbino geometry allows a two-point measurement of the diagonal conductivity σ_{xx} which will be referred to as σ throughout this paper. The two-point measurement is the only possible type of measurement at microwave frequencies. The relation between the conductance $G = I/U$ with current I and voltage U and the conductivity σ is given by $\sigma = G/2\pi \ln(r_2/r_1)$, where $r_2 = 820$ μm is the outer radius of the Corbino ring and $r_1 = 800$ μm is its inner radius.

The sample was used as termination of a standard coaxial line with impedance $Z_0 = 50 \, \Omega$. The deviation of the sample impedance $Z = 1/G$ from Z_0 leads to reflection of a microwave signal fed into
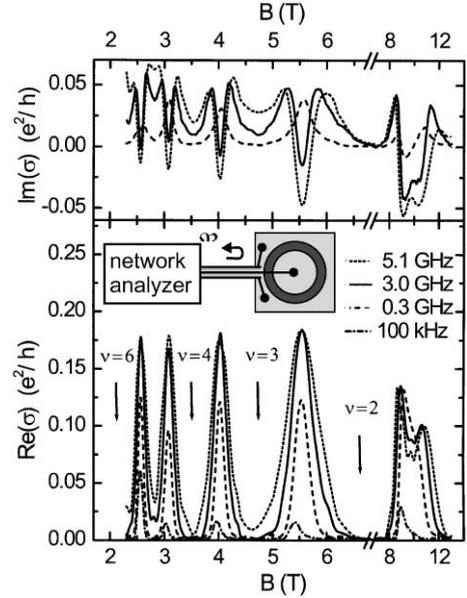

Fig. 1. Real and imaginary part of conductivity for different frequencies.

the line. The reflection coefficient at the end of the line is given by $\mathscr{R}_p = (Z - Z_0)/(Z + Z_0)$. By measuring this reflection coefficient we are able to deduce the conductivity σ. The setup would be most sensitive to changes in σ if the sample impedance $Z = 1/G$ was close to the line impedance $Z_0 = 50 \, \Omega$. This is the reason why we chose such extreme aspect ratio of the Corbino disc yielding $Z \gtrsim 1$ kΩ for the expected conductivity $\sigma \lesssim 0.5e^2/h$. We are still far away from $Z = Z_0$, but a smaller ring width would lead to size effects. Sample and coaxial line were fitted into a dilution refrigerator with base temperature $T_S < 50$ mK. Great care was taken on the thermal sinking of the coaxial line. An important point is a careful characterization of the frequency dependent losses, phase shifts and connector reflections of the coaxial line. With this information we are able to extract the frequency dependent sample reflection coefficient $\mathscr{R}_p$ from the total reflection $\mathscr{R}$ of the line measured with a network analyzer with frequency range 100 kHz–6 GHz. The result of such a measurement of the sample conductivity as a function of the magnetic field is shown in Fig. 1. Our measurement technique naturally gives access to real and imaginary part of σ.
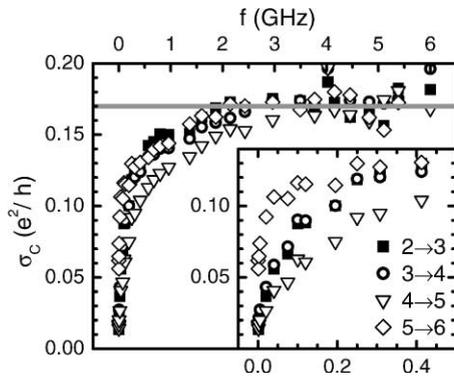
Fig. 2. Peak values of the real part of conductivity for different plateau transitions.

The measured amplitude of the Shubnikov-de Haas (SdH) oscillations of the real part of the conductivity Re(σ) strongly rises from 100 kHz to 300 MHz (Fig. 2). The magnetic field dependence of the imaginary part Im(σ) in this low frequency range is also distinct from the high frequency behaviour and shows SdH oscillation in phase with the real part. This contradicts scaling theory where frequency effects should be negligible for $hf \ll k_B T$. With an electron temperature $T_e \geqslant 100$ mK a marginal frequency dependence for $f < 1$ GHz would follow. The reason for this disagreement lies in the sample geometry: The two point Corbino geometry is sensitive to contact effects. An accumulation or depletion zone along the ohmic contacts leads to edge modes not expected for ideal contacts. Such effects were observed (e.g. in Ref. [6]), and in recent work direct imaging of an edge structure in a Corbino geometry was performed [7]. This edge structure leads to an additional resistance in series with the 2DES and therefore a reduced total DC-conductance of the sample. The DC-transport mechanism from the contacts into the edge structure and further into the undisturbed 2DES is probably governed by tunneling processes. This explains the low values of the critical conductivity, which also depends on the filling factor, measured in most experiments using Corbino geometry.

For AC-transport an additional transport mechanism from contact to undisturbed (bulk) 2DES is opened. This might be capacitive coupling as hinted by Im(σ) in an intermediate frequency range

represented by $f = 300$ MHz in Fig. 1. At sufficiently high frequencies, the additional edge series resistance becomes small compared to the resistance of the bulk 2DES and the conductance measurement yields the true conductivity of the electron system. For our measurement this is true for $f > 2$ GHz. Since we got rid of the disadvantages of Corbino geometry, namely the contact resistance, by applying high frequency, we are left with an advantage in comparison to Hall geometry: Using Corbino geometry, we have direct access to the longitudinal conductivity, while for Hall geometries it is necessary to invert the resistivity tensor with possible errors due to geometry and inhomogeneities.

For high frequencies σ_c scatters around $\sigma_c \approx 0.17 e^2/h$ and shows no further systematic frequency or filling factor dependence (Fig. 2). Also the temperature dependence of σ_c at these high frequencies is found to be negligible, whereas σ_c shows a strong decrease with decreasing temperature at low frequencies. Again this behaviour can be modeled by a strongly temperature dependent edge resistance in series with the intrinsic 2DES resistance.

At high frequencies, the critical conductivity σ_c follows the predictions of scaling theory, but is still significantly lower than the proposed universal value $\sigma_c = 0.5 e^2/h$. It is of the same order of magnitude as the values found by Rokhinson et al. [8] in one of the few experiments without apparent filling factor dependence. Believing in universality one possible explanation for our experimental findings was given by Ruzin et al. [9]. They showed that macroscopic inhomogeneities of the carrier density would lead to a critical conductivity deviating from its universal microscopic value.

Leaving the critical point the next step is an analysis of the transition width Δv between quantum Hall plateaus. Before heading to frequency scaling the first step is to test for temperature scaling. Fig. 3(a) shows the temperature dependence of the plateau transition width Δv plotted on logarithmic scale. The plot is representative for all transitions in the filling factor range $v = 2$–6. The low frequency curves are well described by a power law $\Delta v \propto T^\kappa$ with best fit results $0.39 < \kappa < 0.45$ for different transitions. This result was tested at
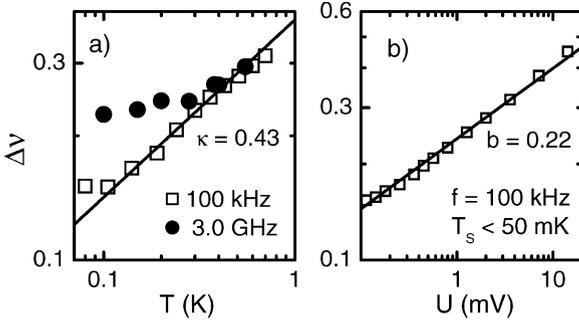
Fig. 3. Plateau transition width for transition $\nu = 3 \to 4$ defined as FWHM of the conductivity peak and scaling analysis of (a) temperature and (b) voltage dependence.

Fig. 4. Scaling analysis of the frequency dependence of the plateau transition width.

different frequencies (100 kHz and 300 MHz) and fits the commonly measured exponent $\kappa = p/2\gamma = 0.43$ which is expected for $\gamma = 2.3$ and $p = 2$. Our data does not fit the linear dependence found in an experiment of Balaban et al. [4]. Using the scaling behaviour of the transition width as low temperature thermometry for the electron system we estimate an electron temperature T_e for a base temperature $T_s < 50$ mK between 100 and 150 mK, slightly dependent on magnetic field.

A second confirmation of scaling behaviour is given by the voltage dependence of $\Delta\nu$ shown in Fig. 3(b): It follows a power law $\Delta\nu \propto U^b$. Interpretation due to a voltage dependent effective temperature $T_e \propto U^a$ with $a = b/\kappa$ leads to $a = 2/(2 + p)$. An exponent $a = 0.5$ equivalent to $p = 2$ leads to $b = 0.22$ which fits our data.

The second data set in Fig. 3(a) with $f = 3$ GHz represents the situation at high frequencies with $\Delta\nu$ defined as the FWHM of the real conductivity $\mathrm{Re}(\sigma)$: At low temperatures, the transition width is no longer temperature dependent, but is determined by the frequency.

In Fig. 4, the frequency dependence of $\Delta\nu$ is plotted for plateau transition $\nu = 2 \to 3$. As shown in the previous section the frequency governs the transition width for $f \geqslant 3$ GHz while for $f \leqslant 1$ GHz the electron temperature leads to saturation. This restricts a conventional scaling analysis to a frequency range $f = 3$–6 GHz. A power law fit $\Delta\nu \propto f^c$ in this range is shown as straight line and leads to an exponent $c = 1/z\gamma = 0.6 \pm 0.1$, which is higher than the expected 0.43 for $z = 1$ and $\gamma \approx 2.3$. For
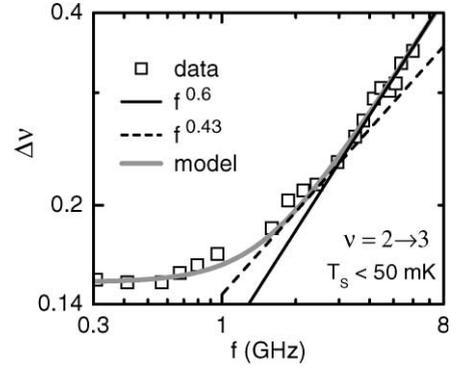
comparison a power law with this exponent is plotted as dashed line. It is clearly less favourable than the higher exponent. To overcome the unsatisfactory small fitting range it is necessary to incorporate both frequency and temperature into the scaling analysis. We chose as an ansatz $L_{\mathrm{eff}}^{-2} = L_\Phi^{-2} + L_f^{-2}$. The motivation is an addition of scattering rates: The power law of the phase coherence length $L_\Phi \propto T^{-p/2}$ follows from $L_\Phi = \sqrt{D/\Gamma_\Phi}$ with the inelastic scattering rate Γ_Φ and diffusion constant D. Interpreting in analogous way $\Gamma_f = D/L_f^2$ as frequency scattering rate the Mathiessen rule $\Gamma = \sum \Gamma_i$ leads to the proposed ansatz. The resulting transition width is

$$\Delta\nu = \Delta\nu_0 \left(\left(\frac{T}{T_0} \right)^p + \left(\frac{f}{f_0} \right)^{2/z} \right)^{1/(2\gamma)}.$$

One of the parameters $\Delta\nu_0$, T_0 and f_0 can be chosen arbitrarily. Here we choose $T_0 = 1$ K and use $\Delta\nu_0$, $p = 2$ and $\gamma = 2.3$ from temperature and voltage scaling. A least square fit with remaining parameters f_0 and z for transitions in the filling factor range $\nu = 2$–6 leads to $z = 0.75 \pm 0.1$ compatible with an exponent $c \approx 0.6$ for pure frequency scaling and deviating from the expected dynamical exponent $z = 1$. The fit is shown as grey line in Fig. 4.

In conclusion, we were able to measure the frequency dependent complex conductivity of a 2DES at quantizing magnetic fields and temperatures $T \leqslant 100$ mK up to 6 GHz. This allows us to overcome contact effects and to measure the critical conductivity at the quantum Hall plateau

transition which deviates from the proposed universal value. Second, we performed a scaling analysis of the plateau transition width and incorporated a two parameter ansatz, taking into account both temperature and frequency. We find significantly different scaling exponents for these two parameters which are so far not understood theoretically.

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