Hopping Conductivity in the Quantum Hall Effect: Revival of Universal Scaling

F. Hohls,* U. Zeitler, and R. J. Haug
Institut für Festkörperphysik, Universität Hannover, Appelstrasse 2, 30167 Hannover, Germany
(Received 16 July 2001; published 2 January 2002)

We have measured the temperature dependence of the conductivity σ_{xx} of a two-dimensional electron system deep into the localized regime of the quantum Hall plateau transition. Using variable-range hopping theory we extract directly the localization length ξ from this experiment. We use our results to study the scaling behavior of ξ as a function of the filling factor distance $|\delta\nu|$ to the critical point of the transition. We find for all samples a power-law behavior $\xi \propto |\delta\nu|^{-\gamma}$ in agreement with the theoretically proposed universal exponent $\gamma = 2.35$.

DOI: 10.1103/PhysRevLett.88.036802 PACS numbers: 73.43.–f, 71.23.An, 71.30.+h, 72.20.My

The prominent feature of the quantum Hall effect (QHE) is the emergence of quantized plateaus in the Hall resistance of a two-dimensional electron system (2DES) reflecting the localization of states at the Fermi edge. A deeper understanding of this fascinating phenomenon is gained from regarding the transition region between adjacent QHE plateaus [1,2]. In this regime the dissipative transport is governed by the delocalized states in the vicinity of the Landau level center. The degree of localization is expressed by the localization length ξ denoting the typical extension of the electron wave function. For any sample with a finite size L theory predicts the conductivity tensor in the plateau transition to follow a scaling function $f\,(L/\xi)$ with a diverging localization length $\xi = |\delta\nu|^{-\gamma}$ [3] where $\delta\nu = \nu - \nu_c$ denotes the filling factor distance to the critical point ν_c of the transition. The critical exponent $\gamma = 2.35 \pm 0.03$ was predicted to be universal, its value determined numerically [1,4–6] and validated by variation of the sample size [7]. Experimentally the scaling of the conductivity near the critical point was verified with great success for different samples by temperature, current, and frequency dependent measurements of the transition width [8–10]. These new parameters introduce effective lengths $L_T \propto T^{p/2}$, $L_I \propto I^{p/(2+p)}$, and $L_f \propto f^{1/z}$ and thereby add additional exponents z and p. These effective lengths replace the physical sample size L in the scaling functions $f(L/\xi)$. Recent experiments extended the focus to the transition from the quantum Hall state to the Hall insulator [11–13]. In these investigations striking similarities to the scaling behavior in the transition between different QH states were observed [14] hinting of the same universality class for both types of transitions [15,16].

In spite of the great success of scaling theory in the QHE there are still some experiments not fitting into the picture of universality. Nonuniversal exponents were observed in the dependence of the transition width on temperature [17], current [18], and frequency [19]. Other experiments seem to contradict scaling theory at all, both for the Hall plateau-insulator transition [20] and the transition between QHE plateaus [21,22].

However, before making conclusions on a general failure of scaling theory it has to be considered that nearly all experiments *do not* measure the localization length ξ directly. Therefore an assumption about the functional form and exponents z and p of the effective length $L_{\mathrm{eff}}(T, I, f)$ has to be made. The nonuniversal scaling exponents deduced from investigating the QHE transition width as a function of T, I, and f then reflect only nonuniversal exponents p and z in L_{eff}. Additionally, the measurements focus mostly on the region close to critical points, where ξ becomes larger than L_{eff}. In this regime, however, the mechanism of electronic transport at nonzero temperature or frequency is not thoroughly understood. Addressing this problem Shimshoni found in a recent theoretical work for the Hall plateau-insulator transition that for $T > 0$ quantum transport occurs only at some distance to the critical point, namely in the regime of hopping conductivity [23]. Therefore, any lack of universal width scaling with T or f does not necessarily allow one to draw conclusions on the behavior of ξ.

In order to avoid such complications, we follow a different approach to scaling. We directly evaluate the localization length in the well understood regime of variable-range hopping (VRH) conductivity [24]. We have recently shown this method to be reliable for frequency dependent measurements [25]. VRH dominates the conductivity at low temperatures, when the localization length becomes much smaller than the effective temperature length L_T. In the QHE regime the VRH conductivity is given as [24,26,27]

$$\sigma_{xx}(T) = \sigma_0 \exp(-\sqrt{T_0/T}\,), \qquad k_B T_0 = C\,\frac{e^2}{4\pi\epsilon\epsilon_0\xi}\,, \tag{1}$$

with a temperature dependent prefactor $\sigma_0 \propto 1/T$. The characteristic temperature T_0 is determined by the Coulomb energy at a length scale given by the localization length ξ, the dimensionless constant C being on the order of unity. With this direct access to $\xi \propto 1/T_0$, it is possible to test for scaling behavior of ξ at the edges of the plateau as long as we are careful enough to stay within the localized regime.

Earlier experiments confirmed the expected temperature dependence of $\sigma_{xx}(T)$ and extracted ξ in the QHE

 0031-9007/02/88(3)/036802(4)$20.00

regime [28,29]. In a recent experiment on the QH plateau-insulator transition VRH conductivity following Eq. (1) was also established to that regime and was used to determine ξ, finding a rough agreement with the prediction of universal scaling [30].

In this paper we show that when analyzing the conductivity deep into the localized regime of the quantum Hall plateau transition we find a clear universal scaling behavior of the localization length $\xi \propto |\delta\nu|^{-\gamma}$ as a function of the filling factor distance $\delta\nu = \nu - \nu_c$ to the critical point at ν_c with a universal $\gamma = 2.35$. Such a universality is observed even in samples where nonuniversal exponents extracted from temperature dependent peak-width scaling are found. Going even further, we show that the conductivities $\sigma_{xx}(T, \delta\nu)$ in the VRH regime for $\delta\nu < 0.3$ can be scaled to a single parameter function of $|\delta\nu|^{\gamma}/T$ as predicted by scaling theory for the critical regime.

The samples used in this work are based on modulation doped GaAs/AlGaAs heterostructures with additional scatterers in the active region of the 2DES [31,32]. The scatterers are provided by doping the GaAs close to the heterojunction with Si or Be, either as a δ layer or a weak homogeneous background. This results in relatively low mobilities of a few 10^2/V s (see Table I). In order to allow a highly sensitive two-point measurement of very low conductivities the samples were patterned into Corbino geometry using contacts fabricated by standard Ni/Au/Ge alloy annealing. Here the conductivity is given by $\sigma_{xx} = (I/2\pi V)\ln(r_2/r_1)$ where $r_1 = 500\ \mu$m and $r_2 = 550\ \mu$m are the inner and the outer radii of the Corbino disk. The samples were mounted onto the cold finger of a dilution refrigerator with a base temperature below 20 mK and positioned into the center of a superconducting solenoid.

The sample conductivity $\sigma_{xx}(B, T)$ as a function of magnetic field B and temperature T was extracted from individually measured I-V characteristics by numerical differentiation in the linear regime close to zero bias. This method allowed us to measure the conductivity accurately in a huge range between $10^{-13}\ \Omega^{-1}$ in the plateau center up to $10^{-5}\ \Omega^{-1}$ in the maximum of the plateau transition.

In Fig. 1(a) the conductivity $\sigma_{xx}(B, T)$ of sample S1 is shown for the transition between the QHE plateaus at $\nu = 1$ and $\nu = 2$. We concentrate our studies on this transition where the spin gap is sufficiently large to exclude activated transport. The critical point ν_c of this transition, listed in Table I, is given by the position of the maximum of $\sigma_{xx}(\nu)$.

Before analyzing the VRH transport we tested the conventional scaling behavior of the plateau transition width. Data for the full width at half maximum $\Delta\nu$ of the peak in σ_{xx} as a function of filling factor $\nu = hn/eB$ are shown in Fig. 1(b) for sample S1. For all investigated samples $\Delta\nu$ follows a power-law behavior $\Delta\nu \propto T^{\kappa}$ down to temperatures below 60 mK. However, as already reported in earlier work on similar samples [17] the critical exponents $\kappa = p/2\gamma$ as presented in Table I deviate considerably from the proposed universal value $\kappa = 0.43$ [8].

Let us now turn to a closer analysis of the VRH regime. As shown in Fig. 1(c) for sample S1 the data fit well the predicted temperature behavior of Eq. (1). We also tested for activated behavior, Coulomb gap with constant σ_0, and Mott hopping $\sigma_{xx} \propto T^{-m}\exp[(T_0/T)^{1/3}]$ for various m, all matching our data worse. From these fits to Eq. (1) we are able to extract the characteristic temperature T_0 in the VRH regime. To stay well inside this localized regime we take into account only filling factors where the low temperature conductivity is at least 2 orders of magnitude below the critical conductivity σ_c.

In Fig. 2 the characteristic temperature T_0 is plotted against the distance $|\delta\nu| = |\nu - \nu_c|$ to the critical point ν_c. The data follows a power-law behavior $T_0 \propto |\delta\nu|^{\gamma}$,

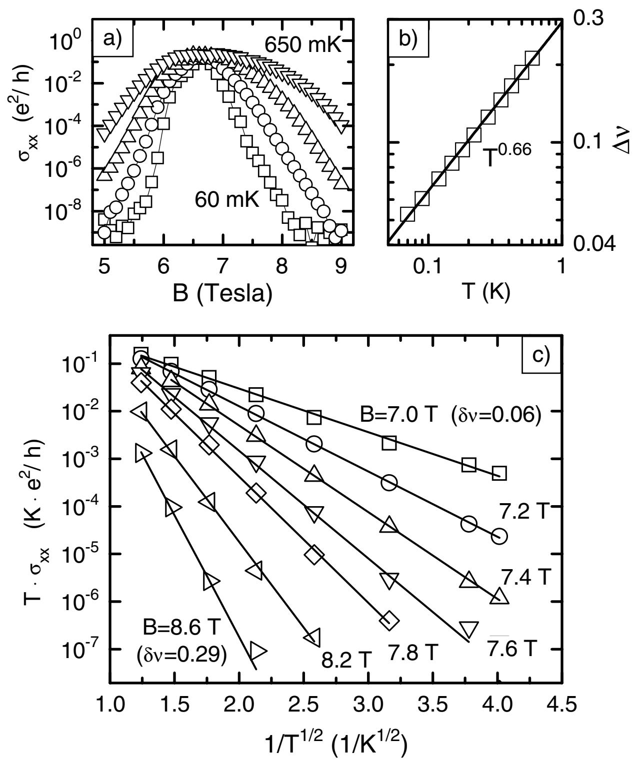

FIG. 1. (a) Conductivity σ_{xx} of sample S1 at different temperatures for the transition between plateaus $\nu = 2$ and $\nu = 1$. (b) Full width at half maximum $\Delta\nu$ of the conductivity peak fitted by a power-law $\Delta\nu \propto T^{\kappa}$. (c) Variable-range hopping fit of σ_{xx} to Eq. (1) with a prefactor $\sigma_0 \propto 1/T$. The axes are rescaled to show a straight line for the prediction of Eq. (1).

TABLE I. Sample characteristics: Type of extra doping, electron density n_e, and mobility μ_e of the 2DES, the critical point ν_c [33], and the width scaling exponent κ for the $\nu = 1 \rightarrow 2$ transition.

Sample	Doping	n_e (10^{15} m^{-2})	μ_e 10^2/V s	ν_c	κ
S1	δ-Be	2.1	2	1.29	0.66 ± 0.02
S2	δ Si	3.2	4	1.62	0.60 ± 0.02
S3	hom. Be	2.4	12	1.52	0.62 ± 0.03

represented by straight lines in this plot, up to distances as large as $|\delta\nu| \approx 0.3$. This demonstrates that the localization length $\xi \propto 1/T_0$ follows a scaling behavior deep into the localized regime. The lines are drawn for the proposed universal scaling exponent $\gamma = 2.35$ and show a nice agreement to the data within the experimental uncertainty. This result strongly supports the proposed universality, being consistent with numerical calculations ([1] and references therein) as well as with size dependent scaling experiments [7] and with a VRH analysis of the frequency dependence of the conductivity [25]. It underlines the fact that our direct scaling analysis in the VRH regime suits much better as an access to the localization length and its scaling behavior than temperature dependent peak-width scaling where no universality of the exponent $\kappa = p/2\gamma$ is observed. As γ is shown to be universal we have to attribute this to the temperature exponent p in $L_T \propto T^{-p/2}$. In fact, for any predictions on p we are still lacking a sufficient knowledge of the temperature dependent transport mechanisms in the metallic regime of the QHE plateau transition.

Now we will estimate the precision of our measurement of the scaling exponent. Linear fits of $\ln(T_0) = \gamma \ln(\delta\nu) + \text{const}$ scatter within 10% around the proposed universal exponent $\gamma = 2.35$. The fitted values are 2.36 ± 0.09 ($\nu > \nu_c$) and 2.20 ± 0.04 ($\nu < \nu_c$) for S1, 2.10 ± 0.03 ($\nu < \nu_c$), and 2.08 ± 0.04 ($\nu < \nu_c$) for S2, and 2.46 ± 0.06 ($\nu > \nu_c$) and 2.50 ± 0.04 ($\nu < \nu_c$) for S3. The given error bars are the pure statistical 1σ intervals resulting from the fit. The total experimental error is presumably larger due to additional systematic errors: The scaling with filling factor distance $\delta\nu$ results from the more fundamental scaling with energy distance $\delta E = E - E_c$ from the critical point. As neither E nor E_c is known, the relation $\delta\nu \propto \delta E$ is used to translate the prediction to measurable quantities. This relation is exactly valid in the case of a flat density of states (DOS) $D(E) = D(E_c)$. Assuming a more realistic Gaussian DOS, a significant deviation from the linear relation $\delta\nu \propto \delta E$ is observed, e.g., 10% for $\delta\nu = 0.3$. Additionally the extra doping with different scatterers and different spatial dopant distributions leads to a more complicated individual asymmetric DOS for each sample [32]. This introduces a further error in $\delta\nu \propto \delta E$ even for smaller $\delta\nu$. The asymmetric DOS also leads to a temperature dependence of the maximum in σ_{xx}, leading to an uncertainty of approximately ± 0.005 in ν_c. Summing up we estimate the uncertainty of the relation $\delta\nu \propto \delta E$ to 5%–10% for $\delta\nu = 0.05, \ldots, 0.3$. Thus the deviation between the data points and the lines in Fig. 2 lies within the experimental uncertainty.

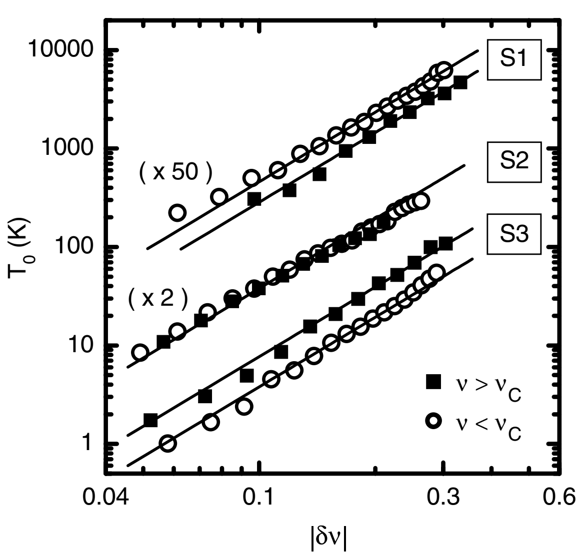

FIG. 2. Characteristic temperature T_0 as a function of the distance $|\delta\nu|$ to the critical point for both sides of the $\nu = 1 \rightarrow 2$ plateau transition on a logarithmic scale. The data for different samples S1, S2, and S3 are shifted for clarity. The vertical size of the symbols equals the statistical 2σ uncertainty of T_0 due to the fits shown in Fig. 1. The lines show the predicted power-law behavior $T_0 \propto |\delta\nu|^\gamma$ with the proposed universal exponent $\gamma = 2.35$.

Until now we used the term scaling in a rather reduced sense as a synonym for a power-law behavior of the localization length ξ. Of course, scaling includes much more, namely the existence of a single parameter scaling function $\sigma_{xx} = f(x)$ with a parameter $x(T, \delta\nu)$. Using our above results in the VRH regime, an appropriate definition is $x = |\delta\nu|^\gamma/T \propto T_0/T$ with the above deduced exponents γ. The postulation of single parameter scaling together with the finding of $\sigma_0 \propto 1/T$ then fixes the prefactor in Eq. (1) to $\sigma_0(\delta\nu, T) = \sigma^*|\delta\nu|^\gamma/T$ with a constant σ^* and yields a scaling of the conductivity

$$\sigma_{xx}\left(\frac{|\delta\nu|^\gamma}{T}\right) = \sigma^* \frac{|\delta\nu|^\gamma}{T} \exp\left(-\sqrt{T^* \frac{|\delta\nu|^\gamma}{T}}\right), \qquad (2)$$

where T^* is constant.

In Fig. 3 we have plotted all the conductivity data $\sigma_{xx}(\nu, T)$ for $\nu > \nu_c$ as a function of a single parameter $|\delta\nu|^\gamma/T$. The values for γ are taken from the power-law fits of T_0. Rescaling the axes in a proper way indeed shows that all experimental points fall onto straight lines as represented by the scaling function in Eq. (2). Therefore, the conductivity in the VRH regime for $\delta\nu < 0.3$ and $\nu > \nu_c$ follows the stringent postulation of single parameter scaling and thus demonstrates the large validity range of the universal scaling phenomenon in the transition between QHE plateaus.

For $\nu < \nu_c$, i.e., approaching the spin gap induced plateau at $\nu = 1$, the single parameter rescaling does not look as good as for approaching the Landau gap at $\nu = 2$, although Fig. 2 showed the same quality of T_0 scaling for both sides of the transition. This discrepancy probably mirrors a more complicated behavior of the prefactor σ_0 in Eq. (1). Obviously, it can no more be written as a simple function $|\delta\nu|^\gamma/T \propto T_0/T$. Such a complicated behavior in this regime is no surprise, taking into account the redistribution of electrons between spin states and the emerging of spin textures as a function of ν at this filling range [34].

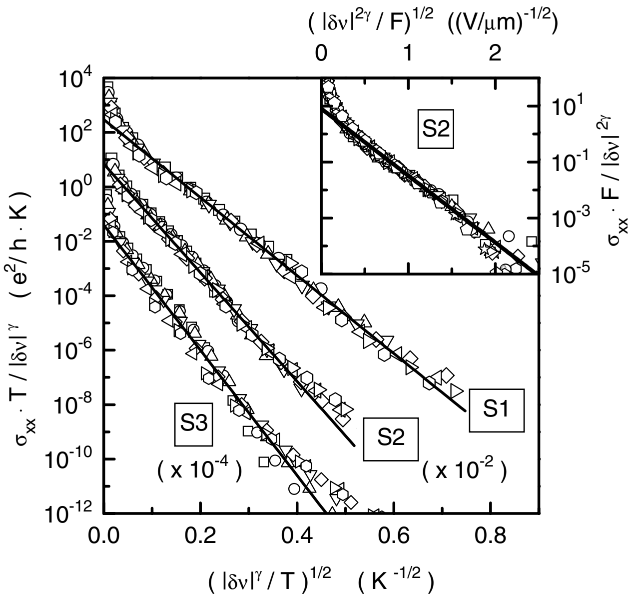

FIG. 3. Rescaling of the conductivity $\sigma_{xx}(T, \nu)$, presented for sample S1 in Fig. 1(c), as function of a single parameter $x = |\delta\nu|^\gamma/T$ for $\nu > \nu_c$ ($T = 60-700$ mK, $|\delta\nu| < 0.3$). Inset: Test of scaling with a single parameter $y = |\delta\nu|^{2\gamma}/F$ for the dependence on electric field ($F = 2-200$ V/m, $\sigma_{xx} \geq 10^{-4}e^2/h$).

In addition to the temperature dependence of σ_{xx} in the VRH regime we have also investigated its dependence on the electric field strength F. In the nonlinear regime at high voltages the effect of F can be interpreted as an effective temperature $T_F = eF\xi/2k_B$ [24]. Using Eq. (1) the conductivity is then rewritten to $\sigma_{xx} = \sigma_0^F(F)\exp(-\sqrt{F_0/F})$ with $F_0 \propto T_0/\xi \propto 1/\xi^2$. Analogous to the single parameter scaling with temperature in the VRH regime, the natural reduced parameter for scaling with electric field is defined by $y(F, \nu) = |\delta\nu|^{2\gamma}/F \propto T_0/T_F$. Applying such an analysis to our data without any additional fit parameter [i.e., using the exponents γ as determined from $T_0(\nu)$ we find for $\nu > \nu_c$ a single parameter scaling for all data in the range $|\delta\nu|^{2\gamma}/F < 4$ μm/V, which is equivalent to the condition $T_F/T_0 > 0.01$ for the ratio of the effective temperature $T_F \propto F\xi$ and the characteristic energy scale $T_0 \propto 1/\xi$. An example of single parameter scaling for sample S2 is shown in the inset of Fig. 3

In conclusion, we have investigated the temperature and electric field dependence of the conductivity in the QHE plateau transition for samples where no universal behavior is found in the conventional temperature dependent peak-width scaling experiments. We find a revival of universality in the VRH regime where the localization length ξ scales as $\xi \propto |\delta\nu|^{-\gamma}$ with an experimentally deduced scaling exponent close to the theoretically expected value $\gamma = 2.35 \pm 0.03$. Even further, we have shown that all the data on the Landau gap side of the transition can be rescaled on a single parameter function over more than 5 orders of magnitude in the conductivity.

We thank F. Evers, B. Huckestein, B. Kramer, F. Kuchar, D.G. Polyakov, L. Schweitzer, and E. Shimshoni for stimulating discussions. The samples were grown by K. Ploog at the Max Planck-Institut für Festkörperforschung.

*Electronic address: hohls@nano.uni-hannover.de

[1] B. Huckestein, Rev. Mod. Phys. **67**, 357 (1995).
[2] S.L. Sondhi *et al.,* Rev. Mod. Phys. **69**, 315 (1997).
[3] A.M.M. Pruisken, Phys. Rev. Lett. **61**, 1297 (1988).
[4] H. Aoki and T. Ando, Phys. Rev. Lett. **54**, 831 (1985).
[5] J.T. Chalker and G.J. Daniell, Phys. Rev. Lett. **61**, 593 (1988).
[6] B. Huckestein and B. Kramer, Phys. Rev. Lett. **64**, 1437 (1990).
[7] S. Koch *et al.,* Phys. Rev. Lett. **67**, 883 (1991).
[8] H.P. Wei *et al.,* Phys. Rev. Lett. **61**, 1294 (1988).
[9] E. Chow *et al.,* Phys. Rev. Lett. **77**, 1143 (1996).
[10] L.W. Engel *et al.,* Phys. Rev. Lett. **71**, 2638 (1993).
[11] D. Shahar *et al.,* Phys. Rev. Lett. **74**, 4511 (1995).
[12] D. Shahar *et al.,* Science **274**, 589 (1996).
[13] M. Hilke *et al.,* Nature (London) **395**, 675 (1998).
[14] D. Shahar *et al.,* Phys. Rev. Lett. **79**, 479 (1997).
[15] W. Pan *et al.,* Phys. Rev. B **55**, 15 431 (1997).
[16] R.T.F. van Schaijk *et al.,* Phys. Rev. Lett. **84**, 1567 (2000).
[17] S. Koch *et al.,* Phys. Rev. B **43**, 6828 (1991).
[18] E. Chow and H.P. Wei, Phys. Rev. B **52**, 13 749 (1995).
[19] F. Hohls *et al.,* Physica (Amsterdam) **298B**, 88 (2001).
[20] D. Shahar *et al.,* Solid State Commun. **107**, 19 (1998).
[21] N.Q. Balaban, U. Meirav, and I. Bar-Joseph, Phys. Rev. Lett. **81**, 4967 (1998).
[22] P.T. Coleridge, Phys. Rev. B **60**, 4493 (1999).
[23] E. Shimshoni, Phys. Rev. B **60**, 10 691 (1999).
[24] D.G. Polyakov and B.I. Shklovskii, Phys. Rev. B **48**, 11 167 (1993); Phys. Rev. Lett. **70**, 3796 (1993).
[25] F. Hohls, U. Zeitler, and R.J. Haug, Phys. Rev. Lett. **86**, 5124 (2001).
[26] G. Ebert *et al.,* Solid State Commun. **45**, 625 (1983).
[27] A. Briggs *et al.,* Phys. Rev. B **27**, 6549 (1983).
[28] S. Koch *et al.,* Semicond. Sci. Technol. **10**, 209 (1995).
[29] M. Furlan, Phys. Rev. B **57**, 14 818 (1998).
[30] S.Q. Murphy *et al.,* Physica (Amsterdam) **6E**, 293 (2000).
[31] K. Ploog, J. Cryst. Growth **81**, 304 (1987).
[32] R.J. Haug *et al.,* Phys. Rev. Lett. **59**, 1349 (1987).
[33] Because of the strong δ doping with Be (Si) the density of states becomes asymmetric and the critical filling factor is shifted from the ideal $\nu_c = 1.5$ to lower (higher) filling factors [32].
[34] S.M. Girvin and A.H. MacDonald, in *Perspectives in Quantum Hall Effects,* edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).