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Variable-range hopping in the quantum Hall regime

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Abstract

We examine the scaling behavior of the transition between adjacent quantum Hall plateaus away from the critical point in the regime of variable-range hopping driven conductivity σ_{xx}. The measured temperature and frequency dependence is used for a direct evaluation of the localization length ξ. We find scaling behavior $\xi \propto |\delta\nu|^{-\gamma}$ up to large filling factor distances $|\delta\nu|$ to the critical point. The scaling exponent $\gamma = 2.3$ agrees with its proposed universal value even for samples which do not show universal behavior within the usual transition-width analysis. This demonstrates the advantage of our variable-range hopping analysis and the robustness of the localization length scaling.

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The understanding of the quantum Hall effect (QHE) is built upon the concept of Landau quantization combined with localization, the latter being characterized by the filling factor dependent localization length $\xi(\nu)$. A small localization length at the Fermi energy results in plateaus in σ_{xy} and vanishing σ_{xx}, while the divergence of ξ at the critical point ν_c in the center of the Landau bands drives the transition between Hall plateaus with a peak in σ_{xx} and transient values of σ_{xy}. The behavior near the critical point is addressed by the scaling theory of localization [1], interpreting the plateau transition as a quantum phase transition and identifying ξ with the correlation length. The theory predicts $\xi \propto |\delta\nu|^{-\gamma}$ with $\delta\nu = \nu - \nu_c$ and an universal exponent $\gamma = 2.3$ determined by numerical calculations. For finite systems of size L one expects scaling relations $\sigma_{\alpha\beta} = f_{\alpha\beta}(\delta\nu L^{1/\gamma})$ with a scaling function $f_{\alpha\beta}$. For non-zero temperature T or frequency f the length L is replaced by an effective length $L_T \propto T^{p/2}$ resp. $L_f \propto f^{1/z}$.

The traditional experimental approach to scaling of the plateau transition is the analysis of the transition-width $\Delta\nu$ of the σ_{xx}-peak, mostly defined as full-width at half-maximum (FWHM), as a function of temperature, current or frequency. Although this experiments were quite successful in establishing scaling theory, they do not give direct access to the scaling exponent γ but result into composed exponents $\kappa = p/2\gamma$ from $\Delta\nu = T^{\kappa}$ and $c = 1/z\gamma$ from $\Delta\nu = f^{c}$. While first measurements seemed to confirm speculations on universal values $\kappa = c = 0.43$ [2], these width scaling exponents were shown to be non-universal [3,4]. Only in one experiment it was possible to determine γ directly by variation of the sample size [6]. Other experiments even yielded width dependences that contradict scaling [5]. Assuming scaling behavior with an universal exponent γ this might show the dissatisfactory understanding of electronic transport within the quasi-metallic regime near

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the critical point and thus the nature of the effective length exponents p and z.

It is preferable to directly access the localization length. A possible approach was proposed by Polyakov and Shklovskii [7]: The remaining conductivity σ_{xx} in the strongly localized regime $\xi \ll L$ at low temperature is governed by temperature or frequency driven variable-range hopping (VRH). The theory of VRH is well understood, and, fitted to measurements of the dependences $\sigma_{xx}(T)$ or $\sigma_{xx}(f)$, it allows an evaluation of the localization length ξ.

Earlier experiments confirmed the expected temperature dependence of $\sigma_{xx}(T)$ and extracted ξ in the QHE-regime [8], but either did not analyze the scaling behavior or were restricted to a small filling factor range close to the quasi-metallic regime. In a very recent experiment we were able to demonstrate universal scaling behavior of ξ in the VRH-regime deep into the plateau [9]. Also very recently we confirmed the proposed linear frequency dependence $\sigma_{xx}(f) \propto f$ in the variable-range hopping regime and evaluated ξ from these measurements [10].

Here, we will demonstrate that the analysis of both the temperature and the frequency dependent VRH-conductivity result into a localization length which follows $\xi \propto |\delta\nu|^{-\gamma}$ with universal exponent $\gamma = 2.3$, although the chosen samples show non-universal exponents of the transition-width scaling with temperature or frequency. In addition for both parameters T and f we extend the range of scaling to a large distance of $|\delta\nu| = 0.3$ to the critical filling factor.

The experiments were performed on two-dimensional electron systems realized in standard AlGaAs/GaAs heterojunctions. Data from two samples are used throughout this paper, the temperature dependence measured with sample S1 and sample S2 utilized for the high frequency measurements. In sample S1 the area of the heterojunction was doped with extra beryllium to achieve a reduced mobility of $\mu = 2\ \mathrm{m^2/V\,s}$ at a density of $n = 2.1 \times 10^{15}\ \mathrm{m^{-2}}$. With no extra doping sample S2 has a moderate mobility of $\mu = 35\ \mathrm{m^2/V\,s}$ and an electron density of $n = 3.3 \times 10^{15}\ \mathrm{m^{-2}}$. The samples were patterned into Corbino geometry by standard Ni/Au/Ge-alloy annealing.

We measured the temperature dependence of the conductivity σ_{xx} for sample S1 at temperatures in the range of $T = 50$–700 mK. We carefully tested the current dependence of σ_{xx} for each analyzed filling factor to avoid heating effects. We concentrate our analysis on the Hall plateau transition between filling factor $\nu = 1$ and 2. The transition-width $\Delta\nu$ (FWHM) of the peak in σ_{xx} follows a power-law behavior $\Delta\nu \propto T^{\kappa}$ with non-universal $\kappa = 0.67$. The regime of variable-range hopping transport starts at a small distance $\delta\nu \approx 0.06$ to the critical point ν_c of the transition. We ruled out activated behavior (inset of Fig. 1) and find good agreement of our data to the expected temperature dependence for VRH [7] given by

$$\sigma_{xx} \propto \frac{\exp(-\sqrt{T_0/T})}{T}, \quad k_B T_0 = \frac{Ce^2}{4\pi\varepsilon\varepsilon_0\xi} \tag{1}$$

with $C \approx 6$ and $\varepsilon \approx 12$. Fits of this equation to our data yielded the filling factor dependence of the localization length $\xi(\nu) \propto 1/T_0(\nu)$, which is shown for $1.0 < \nu < 1.3$ in Fig. 1 as a function of the filling factor distance $|\delta\nu|$ to the critical point $\nu_c = 1.3$. [1] We

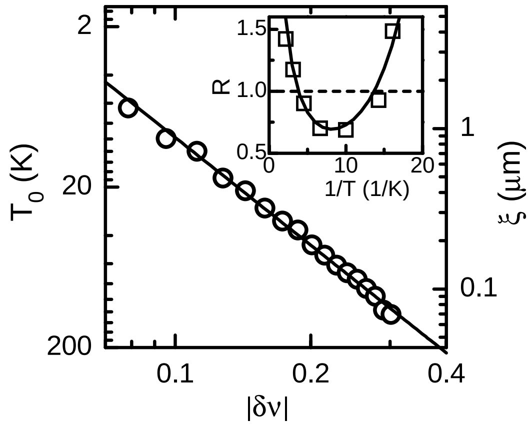

Fig. 1. Characteristic temperature T_0 resp. localization length $\xi \propto 1/T_0$ for sample S1 as a function of the distance $|\delta\nu|$ to the critical point for $1.0 < \nu < \nu_c = 1.3$.[1] The line is a fit to the power law $\xi \propto |\delta\nu|^{-\gamma}$. Inset: Squares show ratio R of the measured σ_{xx} at $B = 7.4$ T to the fit of activated transport. The solid line shows the fit of VRH (Eq. (1)), which represents the data much better than activated transport (dashed line).

[1] Due to the strong Be-doping the density of states becomes asymmetric and the critical filling factor is shifted from the ideal $\nu_c = 1.5$ to a lower value of $\nu_c = 1.3$.

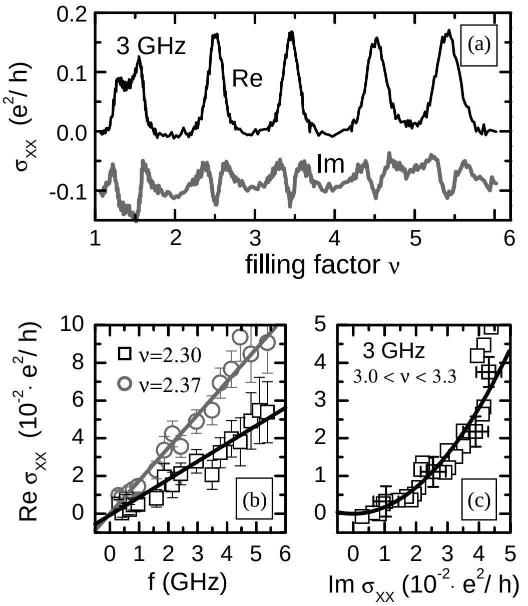

Fig. 2. (a) Real (black line) and imaginary part (grey line, offset by $-0.1\,e^2/h$) of σ_{xx} versus the filling factor for sample S2 at $f=3$ GHz. (b) Linear fit of the frequency dependence for different filling factors. (c) Plot of real versus imaginary part of σ_{xx} for a small filling factor range. The range of the plot was restricted to the frequency driven VRH-regime. As a guide to the eye a parabola is plotted on top of the data.

find a power law $\xi \propto |\delta\nu|^{-\gamma}$ for filling factor distances as large as $|\delta\nu|=0.3$. Although non-universal behavior of the conventional width analysis is observed (see above), the scaling exponent $\gamma = 2.25 \pm 0.06$ fitted to the data agrees well with the proposed universal value of $\gamma = 2.3$.

Now we will focus on the frequency dependence of the conductivity, measured at $T = 100$ mK for $f = 0.001$–6 GHz for sample S2. For technical details of this experiment we refer to Ref. [4]. Our experiment provides us with both the real and imaginary part of σ_{xx}, plotted for S2 at $f = 3$ GHz in Fig. 2(a). The new access to the imaginary part achieved by our experimental technique allows for a new confirmation of scaling behavior, discussed in detail in Ref. [10]. The traditional analysis of the transition-width $\Delta\nu(f)$ shown elsewhere [4] delivers a non-universal frequency scaling exponent $c=0.6\pm0.1$. Here we concentrate on the variable-range hopping regime, which is marked by $\sigma_{xx}(f = 0) \ll \sigma_{xx}(f)$. The frequency dependence of the real part derived by Polyakov and Shklovskii [7] is given by

$$\mathrm{Re}\,\sigma_{xx}(f) = \frac{4\pi^2}{3}\varepsilon\varepsilon_0\xi f \qquad (2)$$

linearly depending on frequency f and the localization length ξ. The linear dependence on f, which we experimentally verified in Ref. [4] for the first time, was confirmed independently by another group [11]. Samples of linear fits to our data are shown in Fig. 2(b). The slope of such a fit directly yields the localization length ξ.

Prior to a further analysis of ξ we want to take a quick look at $\mathrm{Im}\,\sigma_{xx}$ in the VRH-regime. While to our knowledge no analytical expression for the imaginary part of σ_{xx} under quantum Hall condition exists, we can try to compare the case of zero magnetic field. For this case Efros [12] calculated both the real and imaginary part of the conductivity, his relation for the real part differing only by a factor of 2 from Eq. (2). For fixed frequency he derived the relation $\mathrm{Im}\,\sigma_{xx} \propto \mathrm{Re}\,\sigma_{xx}$. In contrast, our data plotted in Fig. 2(c) clearly do not show this linear relation, demonstrating the need for further theoretical understanding of $\mathrm{Im}\,\sigma_{xx}$.

Now we turn back to the localization length ξ. We restrict our analysis of ξ to the range $1.5 < \nu < 4.0$. For higher filling factors the spin splitting is no longer fully resolved, resulting into a significantly enhanced conductivity at $\nu = 5$ compared to other integer filling factors. For $\nu < 1.5$ the σ_{xx}-peak shows a shoulder, which might be caused by an impurity band [13]. As this is probably coupled to additional delocalization, we neither expect nor observe scaling of the localization length for this side of the conductivity peak.

The localization length evaluated from fits of Eq. (2) is shown in Fig. 3. Plotted on a logarithmic scale as a function of the distance $|\delta\nu|$ to the nearest critical point ν_c all but the curve $\nu < \nu_c = 2.5$ agree with a power-law dependence $\xi \propto |\delta\nu|^{-\gamma}$ with universal scaling exponent $\gamma = 2.3$, although the traditional width scaling resulted into a non-universal exponent $c = 1/z\gamma = 0.6 \pm 0.1$. This again demonstrates the advantage of a direct evaluation of the localization length. Similar to our temperature dependent measurement the power law is valid up to about $|\delta\nu| = 0.3$.

The deviation of the data for $\nu < \nu_c = 2.5$ remains an open question. We observe a similar deviation for

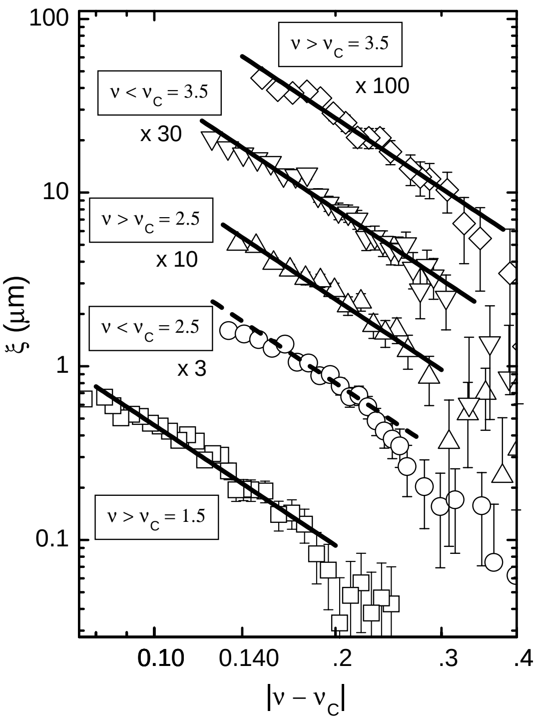

Fig. 3. Localization length ξ for sample S2 as a function of the filling factor difference $\delta\nu = |\nu - \nu_c|$ to the nearest critical point ν_c for the filling factor range $\nu = 1.5$–4. Each curve covers one side of a plateau transition, e.g. the filling factor range $3.5 < \nu < 4.0$ for the top curve. The curves are offset by multiplication with the quoted factors. The straight lines correspond to the expected power law with known universal exponent $\gamma = 2.3$ and prefactors chosen to agree with the data. Except for the dashed line they agree to the data within experimental certitude.

$4.0 < \nu < 4.5$. Thus, this deviation seems to happen when one fills the first electrons to a new, previous empty Landau band.

In conclusion our new approach to scaling, evaluating the localization length ξ from the temperature and frequency dependence of the conductivity in the variable-range hopping regime, confirms the validity of $\xi \propto |\delta\nu|^{-\gamma}$ and recovers universality of the scaling exponent $\gamma = 2.3$ for on first glimpse non-universal samples. The seeming non-universality of the width scaling is probably rooted in an insufficient understanding of transport in the quasi-metallic regime near the critical point. In addition, we extend the scaling regime to a filling factor distance of $|\delta\nu| = 0.3$ to the critical point.

References

[1] B. Huckestein, Rev. Mod. Phys. 67 (1995) 357;
S.L. Sondhi, S.M. Girvin, J.P. Carini, D. Shahar, Rev. Mod. Phys. 69 (1997) 315.
[2] H.P. Wei, D.C. Tsui, M.A. Paalanen, A.M.M. Pruisken, Phys. Rev. Lett. 61 (1988) 1294;
L.W. Engel, D. Shahar, C. Kurdak, D.C. Tsui, Phys. Rev. Lett. 71 (1993) 2638.
[3] S. Koch, R.J. Haug, K.v. Klitzing, K. Ploog, Phys. Rev. B 43 (1991) 6828.
[4] F. Hohls, U. Zeitler, R.J. Haug, K. Pierz, Physica B 298 (2001) 88.
[5] D. Shahar, M. Hilke, C.C. Li, D.C. Tsui, S.L. Sondhi, M. Razeghi, Solid State Commun. 107 (1998) 19;
N.Q. Balaban, U. Meirav, I. Bar-Joseph, Phys. Rev. Lett. 81 (1998) 4967.
[6] S. Koch, R.J. Haug, K.v. Klitzing, K. Ploog, Phys. Rev. Lett. 67 (1991) 883.
[7] D.G. Polyakov, B.I. Shklovskii, Phys. Rev. B 48 (1993) 11 167;
D.G. Polyakov, B.I. Shklovskii, Phys. Rev. Lett. 70 (1993) 3796.
[8] S. Koch, R.J. Haug, K.v. Klitzing, K. Ploog, Semicond. Sci. Technol. 10 (1995) 209;
M. Furlan, Phys. Rev. B 57 (1998) 14 818.
[9] F. Hohls, U. Zeitler, R.J. Haug, cond-mat/0107412, Phys. Rev. Lett., accepted for publication.
[10] F. Hohls, U. Zeitler, R.J. Haug, Phys. Rev. Lett. 86 (2001) 5124.
[11] R.M. Lewis, J.P. Carini, Phys. Rev. B 64 (2001) 073310
[12] A.L. Efros, JETP 62 (1985) 1057.
[13] F. Kuchar, R. Meisels, K. Dybko, B. Kramer, Europhys. Lett. 49 (2000) 480.