Ballistic Heating of a Two-Dimensional Electron System by Phonon Excitation of the Magnetoroton Minimum at $\nu = 1/3$


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A two-dimensional electron system (2DES) is excited with ballistic phonons. Using the 2DES as a thermometer we directly measure the specific heat of the electron system. At a Landau level filling factor $\nu = 1/3$ we deduce a strongly reduced specific heat compared to $\nu = 1/2$ which we can assign to a small but finite density of states in the fractional quantum Hall gap. Furthermore, at $\nu = 1/3$ the phonons are shown to excite finite wave vector magnetorotons across a gap, in excellent agreement with theoretical predictions. [S0031-9007(99)09435-1]

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In a number of recent experiments it has been shown that phonons are a powerful probe of the two-dimensional electron system (2DES) in the fractional quantum Hall (FQH) regime. These experiments include: time averaged phonon absorption [1], temperature dependence of the resistance [2], thermopower [3], and phonon emission measurements [4]. Acoustic phonons can couple efficiently to the 2DES because their typical energies and wavelengths are comparable to the important energy and length scales in the 2DES such as the excitation energy gap, $\Delta$, and the magnetic length, $l_c$.

The low-lying excited states of a 2DES at odd-denominator Landau level filling factors are believed to be collective modes of the incompressible electron liquid. The energy gap related to these excitations has a deep minimum at finite in-plane wave vectors comparable to $l_c^{-1}$ [5,6]. These excitations have been investigated experimentally mainly by Raman scattering techniques [7,8] and by time-averaged [1] and preliminary time-resolved [9] phonon absorption measurements.

The time-resolved phonon absorption experiments reported in this Letter allow us to observe directly the absorption of ballistic phonons by the 2DES. By calibrating the temperature dependence of the 2DES resistance and employing this as a thermometer, the phonon absorption may be used to determine the heat capacity of the 2DES at filling factors $\nu = 1/3$ and $\nu = 1/2$. We show that, at $\nu = 1/3$, ballistic phonons are absorbed by exciting finite wave vector magnetorotons excitations of the highly correlated electron liquid. Because this process is unaffected by disorder on length scales of several phonon wavelengths, we find a magnetoroton gap in excellent agreement with theoretical predictions.

We investigated a 2DES with a sheet density $n = 1.16 \times 10^{15}$ m$^{-2}$ and mobility $\mu = 150$ m$^2$ V$^{-1}$ s$^{-1}$. It was grown on a 2-mm-thick semi-insulating GaAs substrate to allow a sufficiently long phonon time of flight to resolve the response of the system to ballistic phonon pulses with typical pulse lengths of 100 ns. To maximize the sensitivity of the experiment to changes in $\rho_{xx}$, we patterned the device in the form of a meander having a length-to-width ratio, $l/w = 300 (w = 270 \mu$m). The meander covered an area of $5 \times 5$ mm$^2$ and, to avoid contact heating effects, the Ohmic contacts were placed well clear of the active area. The sample was mounted in vacuo on the tail of a dilution refrigerator.

Details of the phonon absorption measurement technique used are given in [10]. However, the special circumstances of this experiment (e.g., the very low temperatures and high sample resistance) required some alterations to the standard technique. The specific experimental arrangement is shown in the inset of Fig. 1. Ballistic phonons with a nonequilibrium temperature $T_h$ were generated by applying short electrical pulses ($\tau = 20–500$ ns) to a metal-film heater on the back face of the substrate opposite the center.

![Diagram](image)

**FIG. 1.** Phonon response showing the variation of the 2DES temperature at $\nu = 1/3$ and $\nu = 1/2$ after a 100 ns pulse of 1.95 K phonons has been emitted by the heater at $t = 0$. The inset shows the sample arrangement for phonon absorption experiments.
of the meander. The heater covered an area $1 \times 1 \text{ mm}^2$, leading to a broad angular distribution ($0^\circ$–$60^\circ$) of phonons incident on the 2DES meander. To avoid substrate heating, the pulse period was kept low (10–100 ms). The 2DES response to the phonon pulse was measured by recording the transient voltage across the meander when applying a 100 nA bias current (low enough to avoid self-heating effects).

To resolve ballistic phonons the response time of the measurement electronics should be (at most) comparable to the phonon pulse length. The response time is limited by the $RC$ time constant of the input resistance of the room temperature low-noise preamplifier ($R = 5 \text{ k}\Omega$) and the coaxial line connecting to the sample. By using a special low capacitance ($C = 30 \text{ pF}$) coaxial line, we were able to achieve a time constant $RC = 150 \text{ ns}$.

In Fig. 1 the responses of the 2DES at filling factors $\nu = 1/3$ and $\nu = 1/2$, for comparison, to a 100-ns pulse of ballistic phonons with $T_e = 1.95 \text{ K}$ are shown: 0.4 $\mu$s after the phonon emission by the heater, the temperature of the 2DES, $T_e$, increases slightly from its equilibrium value $T_0 = 130 \text{ mK}$. This can be assigned to the interaction of the 2DES with longitudinal acoustic (LA) phonons having a time of flight $\tau_{\text{LA}} = 0.4 \mu$s through the 2-mm-thick GaAs substrate. After the time of flight of transverse acoustic (TA) phonons directly traversing the substrate, $\tau_{\text{TA}} = 0.6 \mu$s, a strong increase in $T_e$ is observed, followed by a cooling for $t > 1.2 \mu$s, the time when the last TA phonons (traveling along the [111] direction) hit the corners of the meander structure. Also higher order ballistic TA phonon signals are observed at $3\tau_{\text{TA}} = 1.8 \mu$s and $5\tau_{\text{TA}} = 3 \mu$s proving the ballistic propagation of phonons even over distances $d \geq 10 \text{ mm}$ and the dominance of the TA phonons compared to LA phonons in the absorption process.

It takes a few tens of $\mu$s for all the hot phonons to thermalize via inelastic scattering processes inside the substrate leading to an increase in the background substrate temperature, $T_b$. The electrons reach thermal equilibrium with $T_b$ after thermal relaxation of the hot phonon-excited 2DES into the substrate. $T_b$ is determined experimentally by measuring $T_e$ at long times, $t = 40 \mu$s. It is in good agreement with its expected value calculated from the total energy dissipated in the heater by the phonon pulse and the total specific heat of the substrate. This confirms experimentally the reliability of the thermometry. Finally, with a time constant $t = C_c/\Lambda = 0.5 \text{ ms}$ ($\Lambda$ is the thermal conductance from the sample to the heat sink, mainly via bonded Al wires) the substrate slowly cools to its initial base temperature $T_0 = 130 \text{ mK}$.

The principles of the response of the 2DES to ballistic phonons at $\nu = 1/2$ and $\nu = 1/3$ are quite similar. However, for the given parameters $T_b = 1.95 \text{ K}$ and $\tau = 100 \text{ ns}$, the increase of $T_e$ due to phonon absorption at $\nu = 1/3$ is much stronger than at $\nu = 1/2$ where the shape of the signal is dominated by the background heating of the substrate.

We will first concentrate on behavior at the fractional filling factor $\nu = 1/3$. When the phonon pulse reaches the 2DES, a small proportion of the incident phonon energy, $dE_{\text{ph}} = r(T_e)P_h dt$, is absorbed in a time interval $dt$. Here $P_h$ is the power dissipated from the heater in form of ballistic phonons and $r(T_e)$ is the relative number of phonons absorbed by the 2DES. At $\nu = 1/3$ the phonons are absorbed by exciting finite wave vector excitations around the magnetoroton minimum. However, these chargeless excitations cannot directly contribute to an increased resistance. Nevertheless, provided they decay fast enough inside the 2DES, the energy absorbed will manifest itself as an increase in the electron temperature from its equilibrium value $T_0$ to a nonequilibrium temperature $T_1$. Integrating over the total pulse length $\tau$, the final temperature, $T_1$, of the 2DES is given by

$$\int_{T_0}^{T_1} \frac{C(T_e)}{r(T_e)} dT_e = P_h \tau,$$

where $C(T_e)$ is the total specific heat of the 2DES.

We can determine $C(T_e)$ experimentally from phonon absorption experiments using Eq. (1). The relative proportion of phonons absorbed, $r(T_e)$, is independent of $T_e$ at low enough temperature $T_e \ll \Delta/k_B$ ($\Delta$ is the excitation gap) when the quasiparticle ground state of the 2DES is nearly full and the excited states are nearly empty. Therefore, for the temperature range considered, we can regard $r(T_e) = r_0$ as constant (within an accuracy of $\exp(-\Delta/k_B) \ll 1$).

For a constant power $P_h$, corresponding to $T_h = 1.95 \text{ K}$, we have measured the variation of the ballistic heating $T_1$ for different base temperatures $T_0$ and extracted the relative specific heat $\delta(T) := C(T)/C(0)$ of the 2DES with respect to a free 2DES. Here $C(0) = \pi \hbar \omega_m^2 T_e/6\hbar^2$ is the specific heat for electrons in GaAs with an effective mass, $m^*$. $\Lambda$ is the total surface area of the 2DES. As shown in Fig. 2, the relative specific heat, $\delta(T)$, normalized to the ratio of phonons absorbed, $r_0$, is constant but finite for $T_e < 300 \text{ mK}$. This implies a finite density of states (DOS) in the energy gap probably due to spin excitations, sample inhomogeneity, and edge states. Although this finite DOS is not measured in simple transport experiments, where only the extended DOS at the Fermi level enters, it does contribute to the specific heat even at the lowest carrier temperatures.

For higher temperatures, higher order terms in $C_c(T)$ become important. The experimental data can be well reproduced by an empirical fit $C_c(T) = \alpha T + \beta T^7$, see Fig. 2. Using a simple Sommerfeld expansion [11] for the specific heat this results in a DOS at $\nu = 1/3$ being very flat around $E_F$ and strongly increasing with energy as soon as $|E - E_F|/k_B > 0.5 \text{ K}$ as sketched in the inset of Fig. 2. This is reflected in the fact that the specific heat of
the 2DES at \( \nu = 1/3 \) starts to increase greatly when \( T > 0.5 \) K, as observed experimentally. From the experimental data available up to \( T = 0.8 \) K we can estimate that the specific heat in the \( \nu = 1/3 \) minimum, and, therefore, the quasiparticle DOS at \( \nu = 1/3 \) is reduced by at least 1 order of magnitude compared to the DOS at \( \nu = 1/2 \) where no magnetoquantum oscillations occur.

Knowing the specific heat of the 2DES and using Eq. (1) we can now evaluate the energy absorbed from ballistic phonons as a function of the phonon temperature at fractional Landau level filling \( \nu = 1/3 \). We regard the energy absorption as an indirect consequence of the excitation of the 2DES across the magnetoroton gap \( \Delta \) [1,5,6]. The fast (<50 ns) decay of the excitations inside the 2DES, i.e., without emission of high energy phonons (as will be demonstrated later on), eventually heats the 2DES to a temperature \( T_1 \).

As shown in Fig. 3, the total energy absorbed, \( E_{\text{abs}}(T_h) \), normalized to its value \( E_0 \) for \( T_h = 1.95 \) K and \( \tau = 100 \) ns, increases exponentially when the heater temperature \( T_h \) is increased, inversely proportional to \( \exp(\Delta/k_BT) - 1 \). This increase can be directly related to phonon excitations across a magnetoroton gap, \( \Delta \), experimentally determined as \( \Delta = 0.035(5) \). This value is twice as high as the gap deduced from transport experiments in the same sample. In contrast to transport experiments, our technique is not sensitive to disorder on length scales longer than a few phonon wavelength.

Therefore we are able to measure the gap unaffected by the disorder in the 2DES.

Theory predicts that for an infinitely thin 2DES, the magnetoroton gap is \( \Delta = C_3 e^2/4\pi \epsilon \epsilon_0 l_c \). Here \( l_c = (\hbar/eB)^{1/2} \) is the magnetic length at \( \nu = 1/3 \) and \( C_3 = 0.063(3) \) [6]. Its value is reduced by finite thickness effects [12]. Using the experimentally determined thickness of the 2DES, \( a_0 = 6 \pm 1 \) nm (see below), \( C_3 \) is reduced by a factor 0.55(5) and becomes \( C_3^{\text{red}} = 0.035(5) \) which is in excellent agreement with the experimentally found value \( C_3^{\text{exp}} = 0.036(5) \).

In order to demonstrate excitation across an energy gap even more clearly we have plotted the relative energy absorbed by the 2DES, \( x_{\text{rel}} = E_{\text{abs}}/E_{\text{ph}} \), arbitrarily normalized, as a function of the heater temperature in the inset of Fig. 3. It peaks at a temperature \( T_p = \Delta/4k_B \) where the dominant phonon energy for energy transfer coincides with the magnetoroton gap. The factor of 4 in the value of \( T_p \) suggests that deformation coupling is dominating the energy absorption mechanism.

For \( \nu = 1/2 \) a very pronounced maximum in \( x_{\text{rel}} \) occurs at \( T = 1.1 \) K; see inset in Fig. 3. However, it has a quite different origin than the flat maximum at \( \nu = 1/3 \). It can be attributed to the momentum restriction for phonon-electron coupling due the finite thickness \( a_0 \) of the 2DES. The presence of the maximum in \( x_{\text{rel}}(T_h) \) at \( \nu = 1/2 \) implies that only phonons up to the cutoff can couple to the 2DES. From the position of the \( 1/a_0 \) cutoff

FIG. 2. Relative specific heat of the 2DES at \( \nu = 1/3 \) normalized to the ratio of phonons absorbed as deduced from phonon absorption experiments. The solid line shows a fit taking into account only the \( T \) and \( T^3 \) terms in the Sommerfeld expansion of the specific heat. The inset sketches the CF DOS at \( \nu = 1/3 \) as extracted from the specific heat data.

FIG. 3. Dependence of the energy absorbed by the 2DES on the heater temperature \( T_h \). The solid line shows the fitted behavior for excitation across a magnetoroton gap \( \Delta \). In the inset the relative amount of the total phonon energy as a function of \( T_h \) is shown and compared to the behavior at \( \nu = 1/2 \).
we deduce $a_0 = 6 \pm 1$ nm. Experiments to be reported elsewhere on different samples with different incident angles of the phonons on the 2DES at filling factors 1/2 and 3/2 confirm this scenario of the importance of the $1/a_0$ cutoff and enable us to deduce a similar $a_0$ [13].

In contrast, at $\nu = 1/3$, the excitations occur only when the momentum and energy of the phonons match with the magnetoroton minimum where the structure factor for phonon absorption is maximized. The relative energy absorbed is dependent only on the number of these particular phonons, and, as observed, no heater temperature dependent cutoff exists. This is significant, as if the phonon absorption took place over a wide range of wave vectors it would be expected that the low wave vector phonons would dominate the absorption process due to the strength of the $1/a_0$ cutoff. The absence of this cutoff is strong evidence for the absorption of phonons across a finite energy gap as expected in the fractional quantum Hall liquid.

Final experimental evidence for the direct excitation of magnetorotons by ballistic phonons and an indirect heating of the 2DES due to the decay of these excitations inside the 2DES can been seen in the dependence of the phonon absorption on the length of the heater pulse. We will first concentrate again on the behavior at $\nu = 1/2$ where no magnetorotons exist. All phonon energies up to the cutoff can be absorbed but also emitted by the 2DES. This leads to a saturation of the total energy absorbed as a function of the pulse length $\tau$. In saturation the rate of phonons absorbed and emitted by the 2DES are equal and consequently the 2DES does not absorb any net energy any more even if the phonon pulse is present for longer times. In Fig. 4 this behavior is plotted and compared to a saturation behavior $E_{\text{abs}} \propto 1 - \exp(-\tau/\tau_s)$ with a saturation time $\tau_s = 70$ ns.

The situation is again totally different at $\nu = 1/3$. Here the total energy absorbed by the 2DES increases proportionally with the pulse length with no saturation observed even at times as long as 300 ns, see again Fig. 4. We can conclude that the excited magnetorotons do not come into equilibrium with the ballistic phonons by emitting phonons of the same energy and wave vector on time scales $t > 1 \mu$s. Rather they decay by heating the 2DES internally without phonon emission into the substrate.

In conclusion, the direct absorption of ballistic phonons by the magnetoroton minimum at filling factor $\nu = 1/3$ has been measured for the first time. Phonons have been shown to probe a well-defined finite wave vector in the excitation spectrum. The relaxation of the excited quasiparticles inside the 2DES was used to deduce its specific heat at $\nu = 1/3$. From the dependence of the phonon absorption on the heater temperature we deduced a magnetoroton gap in excellent agreement with theoretical predictions.

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